XVIII International Conference on Water Resources
CMWR 2010
J. Carrera (Ed)
CIMNE, Barcelona 2010

A WELL-BALANCED SPECTRAL VOLUME MODEL FOR CONSTITUENTS TRANSPORT IN ONE-DIMENSIONAL FLOWS

Luca Cozzolino[*], Renata Della Morte[*], Carmine Covelli[†] and Domenico Pianese[†]

[*] Università degli Studi di Napoli Parthenope (UNINAV), Facoltà di Ingegneria
Centro Direzionale di Napoli, 80142 Napoli, Italy
e-mail: luca.cozzolino@uniparthenope.it, renata.dellamorte@uniparthenope.it
web page: http://www.uniparthenope.it

[†] Università degli Studi di Napoli Federico II (UNINA), Facoltà di Ingegneria
Via Claudio 21, 80125 Napoli, Italy
e-mail: carcovel@unina.it, pianese@unina.it, web page: http://www.unina.it

Summary. Numerous finite volume models for the solution of the Shallow-water equations, coupled with the passive transport of a constituent, second-order accurate in time and space, are available in literature, based on different approaches. Recently, numerous methods have been adopted for the calculation of diffusive fluxes in the context of Spectral Volume method, namely the LSV approach and the Penalty SV by Sun and Wang[1], the Penalty SV approach by Kannan and Wang[2]. These methods exhibit one or more of the following problems: lack of symmetry, lack of compactness, sub-optimal order of convergence. Starting from these considerations, in this paper we present a numerical model for the solution of the one-dimensional Shallow-water Equations coupled with the equation for the passive transport of dissolved substances, which takes into account also the diffusion of constituents. The numerical model presented in this paper, which is high-order accurate far from discontinuities of the flow field, is based on the Spectral Volume Method, and applies the HLLC approximate Riemann solver to evaluate the advective fluxes at the interfaces between the spectral cells. In order to ensure the C-Property, the source terms are upwinded at the interfaces, after a so-called "hydrostatic reconstruction". The diffusive fluxes are calculated using a novel approach, called Derivative Recovery Spectral Volume (DRSV), which is linked to the Derivative Recovery Method[3] and to the Direct Discontinuous Galerkin[4], recently introduced for the diffusive fluxes calculation in Runge-Kutta Discontinuous Galerkin methods (RKDG). The DRSV exhibits good properties, namely high-order accuracy, local symmetry and compactness of the numerical stencil. A number of preliminary numerical experiments are reported, showing the promising capabilities of the method.

[1] Y. Sun and Z.J. Wang, "Formulations and analysis of the spectral volume method for the diffusion equation", *Comm. Num. Meth. Engrg.*, **20**, 927-937 (2004).
[2] R. Kannan and Z.J. Wang, "A study of viscous flux formulations for a p-multigrid Spectral Volume Navier Stokes solver", *J. Sci. Comp.*, **41**, 165-199 (2009).
[3] M. van Raalte and B. van Leer, "Bilinear forms for the recovery-based Discontinuous Galerkin Method for the diffusion", *Comm. Comp. Phys.*, **5**, 683-693 (2009).
[4] H. Liu and J. Yan, "The Direct Discontinuous Galerkin (DDG) methods for diffusion problems", SIAM *J. Num. Anal.*, **47**, 675-698 (2009).