

A UNIFIED MULTILEVEL FRAMEWORK OF UPSCALING AND DOMAIN DECOMPOSITION

Andreas Sandvin^{*†}, Jan M. Nordbotten^{†*} and Ivar Aavatsmark^{*}

^{*} Centre for Integrated Petroleum Research (CIPR)
Allegt. 41, 4. etg 5007 Bergen, Norway
e-mail: andreas.sandvin@uni.no, web page: <http://www.cipr.uni.no>

[†] University of Bergen, Department of Mathematics
Johannes Brunsgate 12, 5008 Bergen, Norway
web page: <http://www.uib.no/math>

Summary. The heterogeneous structure of porous rocks may have large variations on different scales. Reservoir simulations (i.e. groundwater flow, CO₂ storage, oil recovery) often involve large spatial scales, where upscaling needs to be applied. Multiscale methods capture the fine-scale variability in coarse scale basis functions, thereby constructing reduced coarse scale problems.

While most multiscale methods are based on a geometric upscaling of fine-scale information, they do not generalize from two- to multi-level methods, or arbitrary geometries and dimensions. This is due to the choice of local boundary conditions for the basis-function problems.

We apply multiscale control-volume methods as preconditioners for a class of mass-conservative domain-decomposition (MCDD) methods, where the local basis-function problems may be formulated as local Schur complement systems. We introduce an algebraic framework, based on probing, for constructing mass-conservative operators on a multiple of coarse scales. These operators may further be applied as coarse spaces for additive Schwartz preconditioners. By applying different local approximations to the Schur complement system based on a careful choice of probing vectors, we show how the MCDD preconditioners can be both efficient preconditioners for iterative methods or accurate upscaling techniques for the heterogeneous elliptic problem. Our results show that the probing technique yield improved approximation properties compared with the reduced boundary condition commonly applied with multiscale control-volume methods.