



CFSI, Ibiza, 5 May 2006

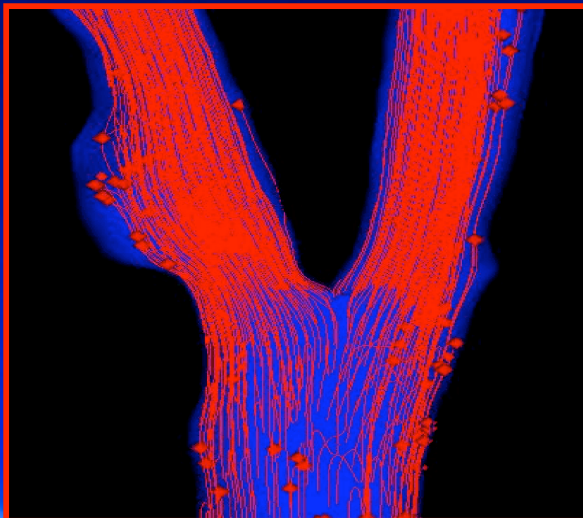


Alfio Quarteroni

EPFL, Lausanne

and MOX, Politecnico di Milano

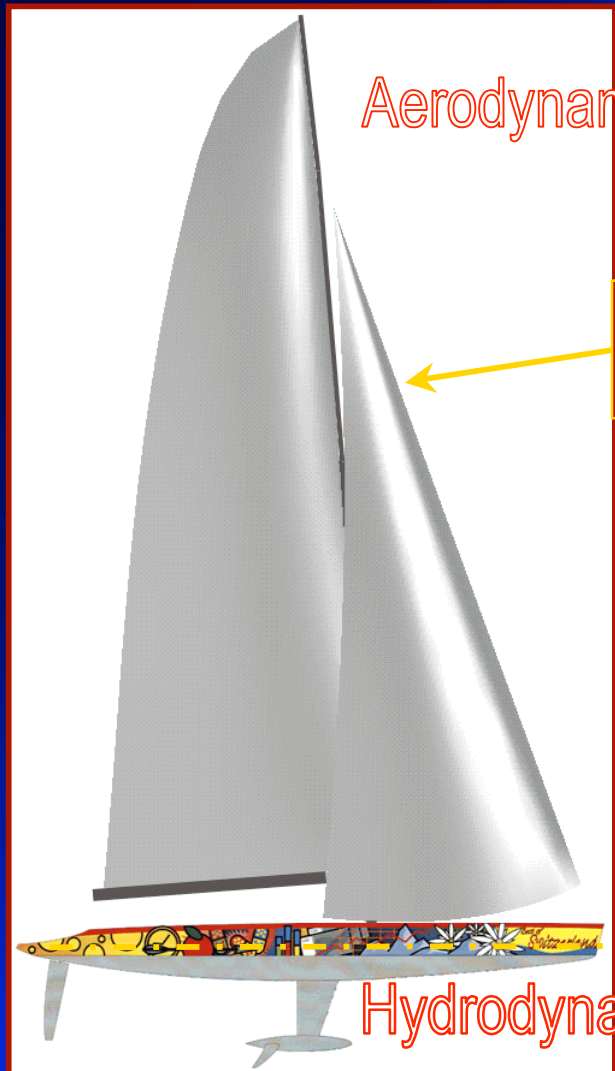
**FLUID-STRUCTURE INTERACTION**  
**TWO APPLICATIONS and SOME REMARKS**



Acknowledgments:  
D. Detomi, G. Fourestey,  
N. Parolini, A. Quaini



# Application 1: Aero-Hydro Dynamics and FSI

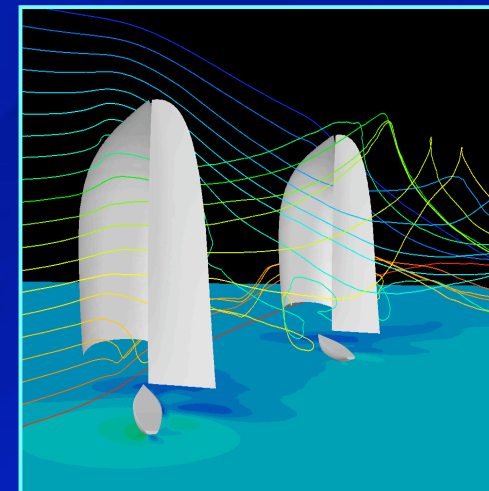


Aerodynamics

aero-elasticity of flexible sails (FSI)

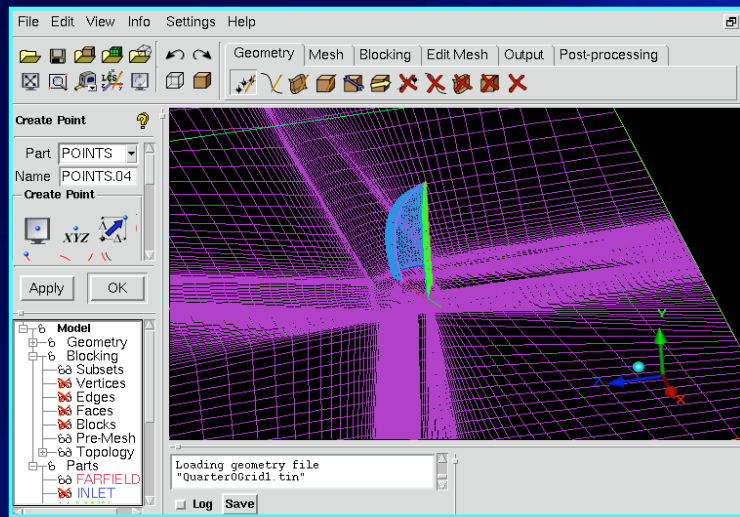
Free Surface

Hydrodynamics

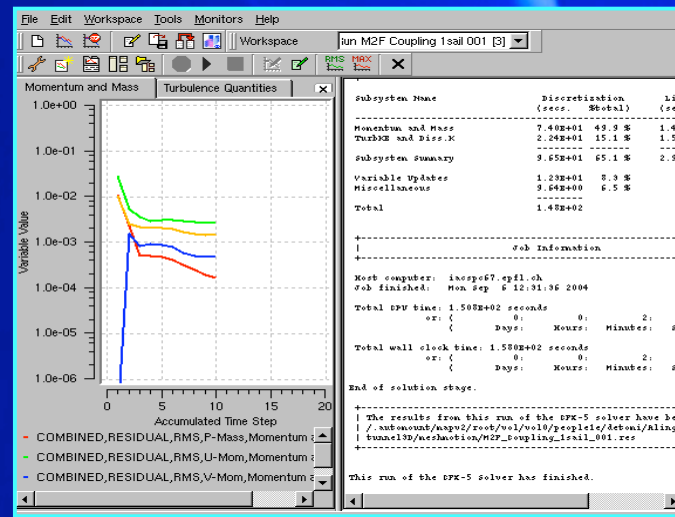




# FSI for Sails: Steady state analysis



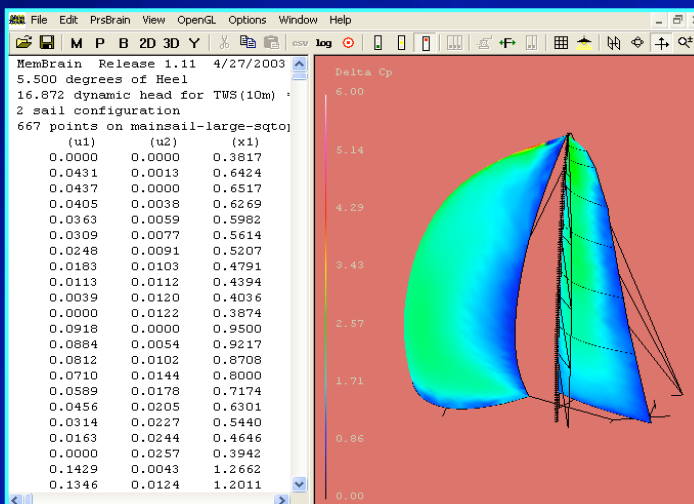
Fluid solver  
(Needs mesh -  
Velocity is null)



Mesher



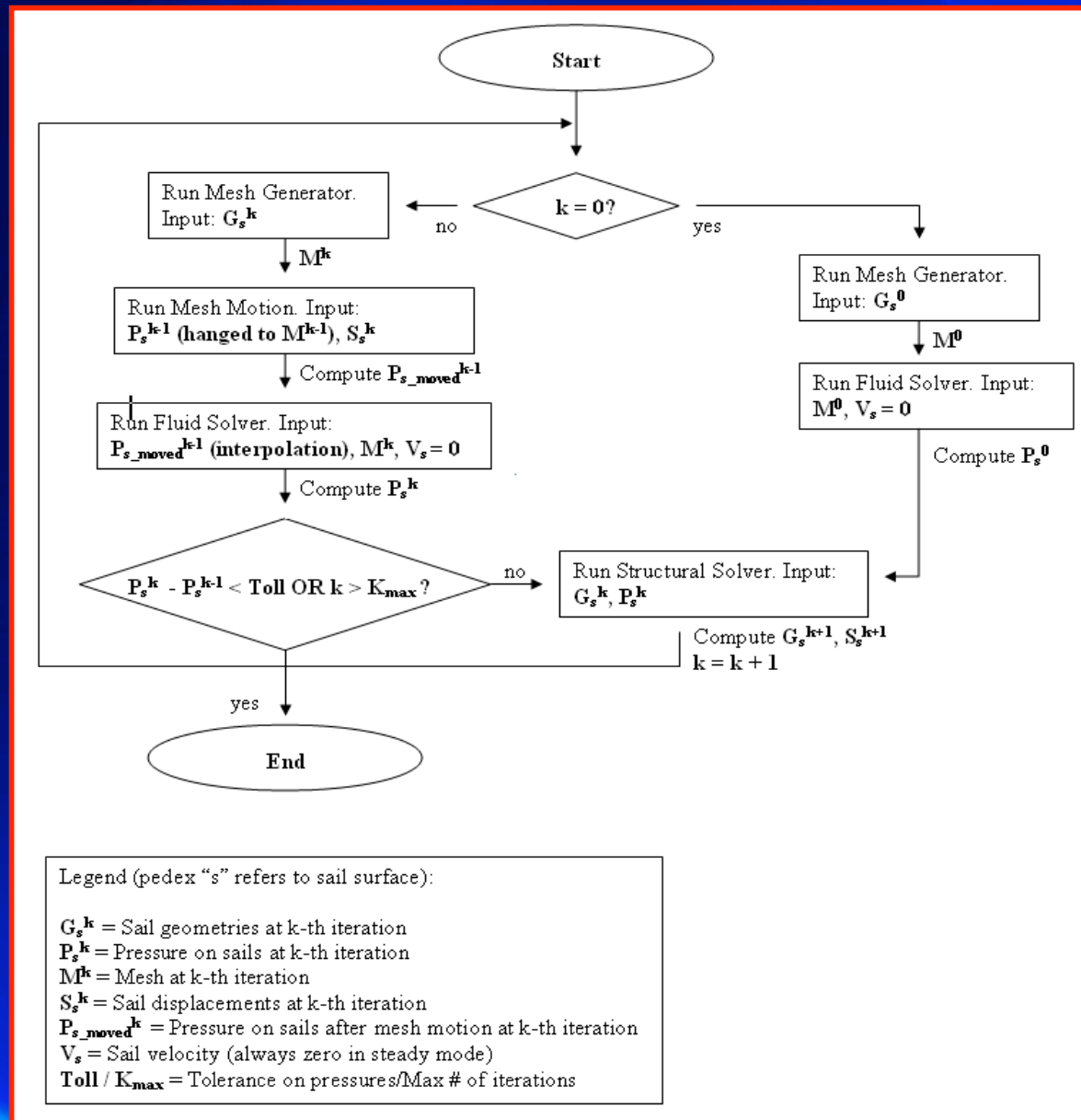
Windows XP → Linux



Structural Solver  
(Needs forces)

Linux → Windows XP

# Fluid-Structure Coupling Scheme



# Equations for sails and sail-fluid coupling

## Elastodynamics equations (small strains, large displacements)

$\psi(x, t)$  : deformation map

$$\rho_s \frac{\partial^2 \psi}{\partial t^2} - \operatorname{div} (\nabla \psi \Sigma) = f_s \quad \text{on } S(0), t > 0$$

$\Sigma = \lambda_s \operatorname{Tr} E I + 2\mu_s E$  : Second Piola-Kirchhoff stress tensor

$E = \frac{1}{2} (\nabla \psi^T \nabla \psi - I)$  : Green-S<sup>t</sup> Venant strain tensor

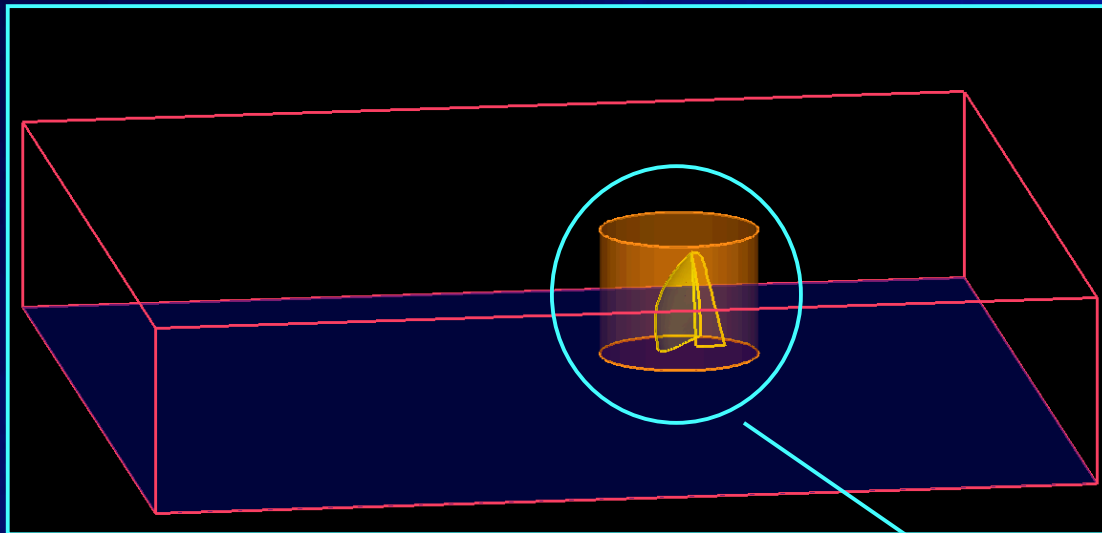
## Coupling conditions (normal stresses and particle velocities must agree)

$$\left\{ \begin{array}{l} \nabla \psi \Sigma n(0) = T(u, p) \circ \psi \operatorname{Cof} (\nabla \psi) n(0) \\ \frac{\partial \eta}{\partial t} = u \circ \psi \end{array} \right. \quad \text{on } \Gamma(0)$$

$\eta(x, t) = \psi(x, t) - x$  : displacement vector field

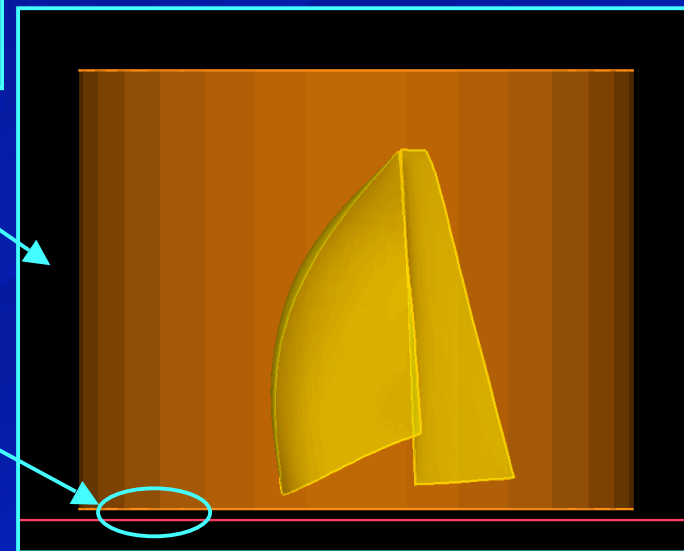


# Domain description



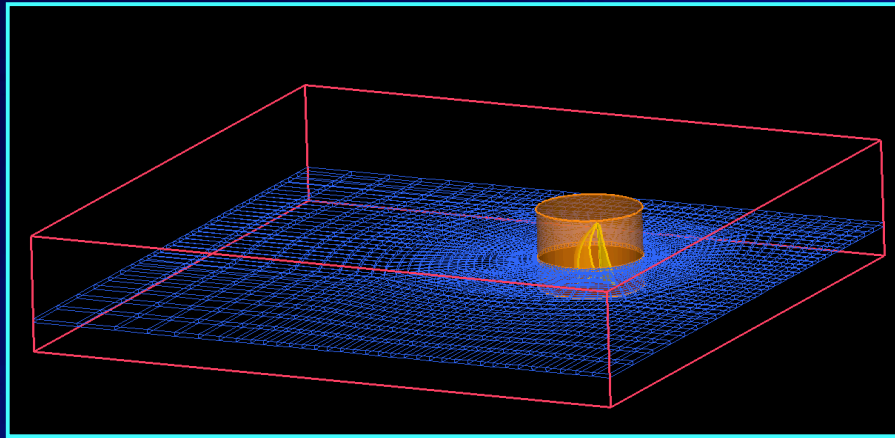
The fluid domain is split in two subdomains, an inner cylinder containing the two sails and a far field region

A boundary layer mesh is created by refining the hexahedral grid within cylinder and sea

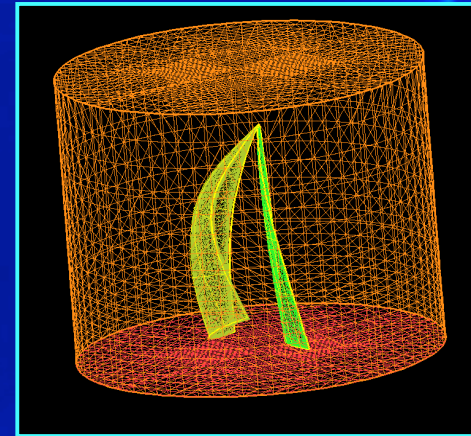


# Meshes

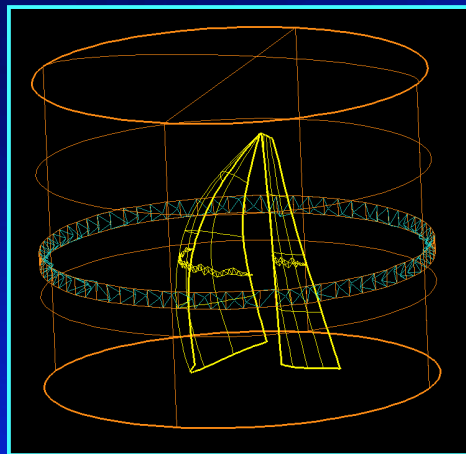
Hexahedra inside the far field



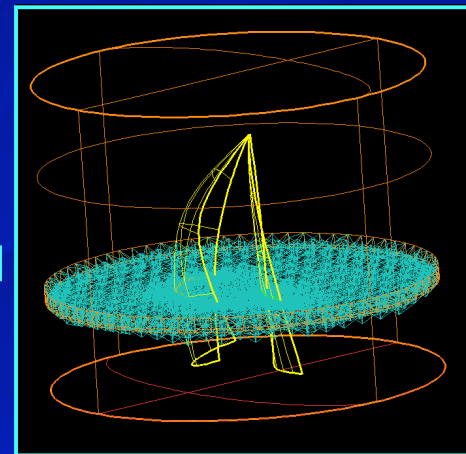
Quad → Tri on cylinder surface



Pyramids to connect the two meshes

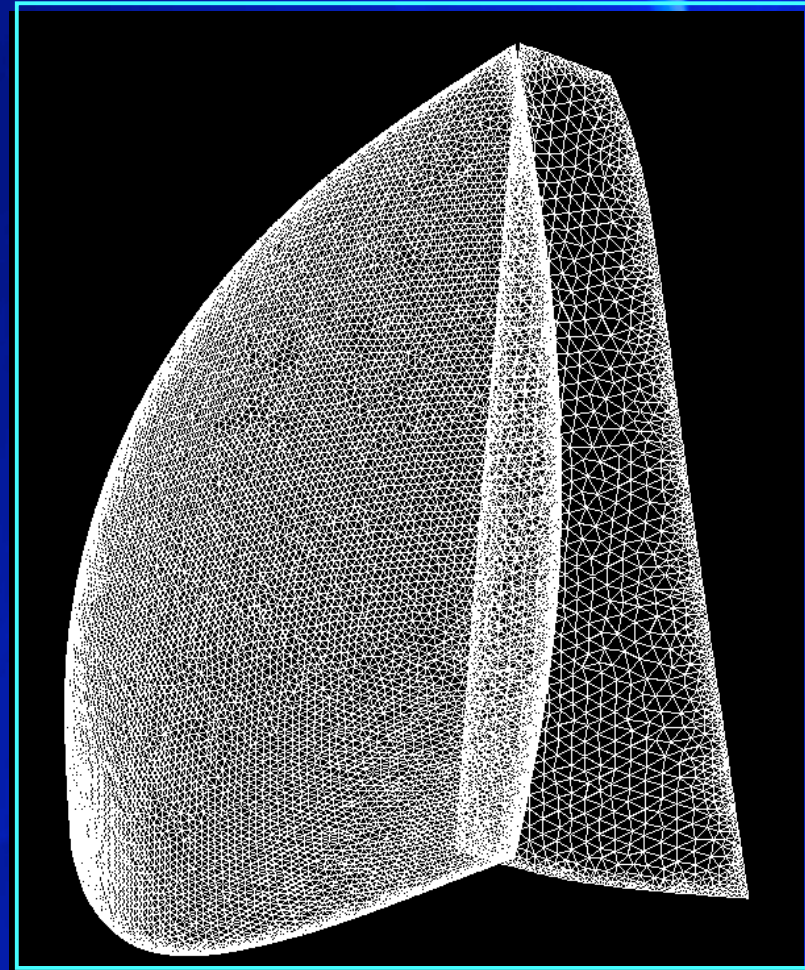


Tetrahedra inside the inner cylinder



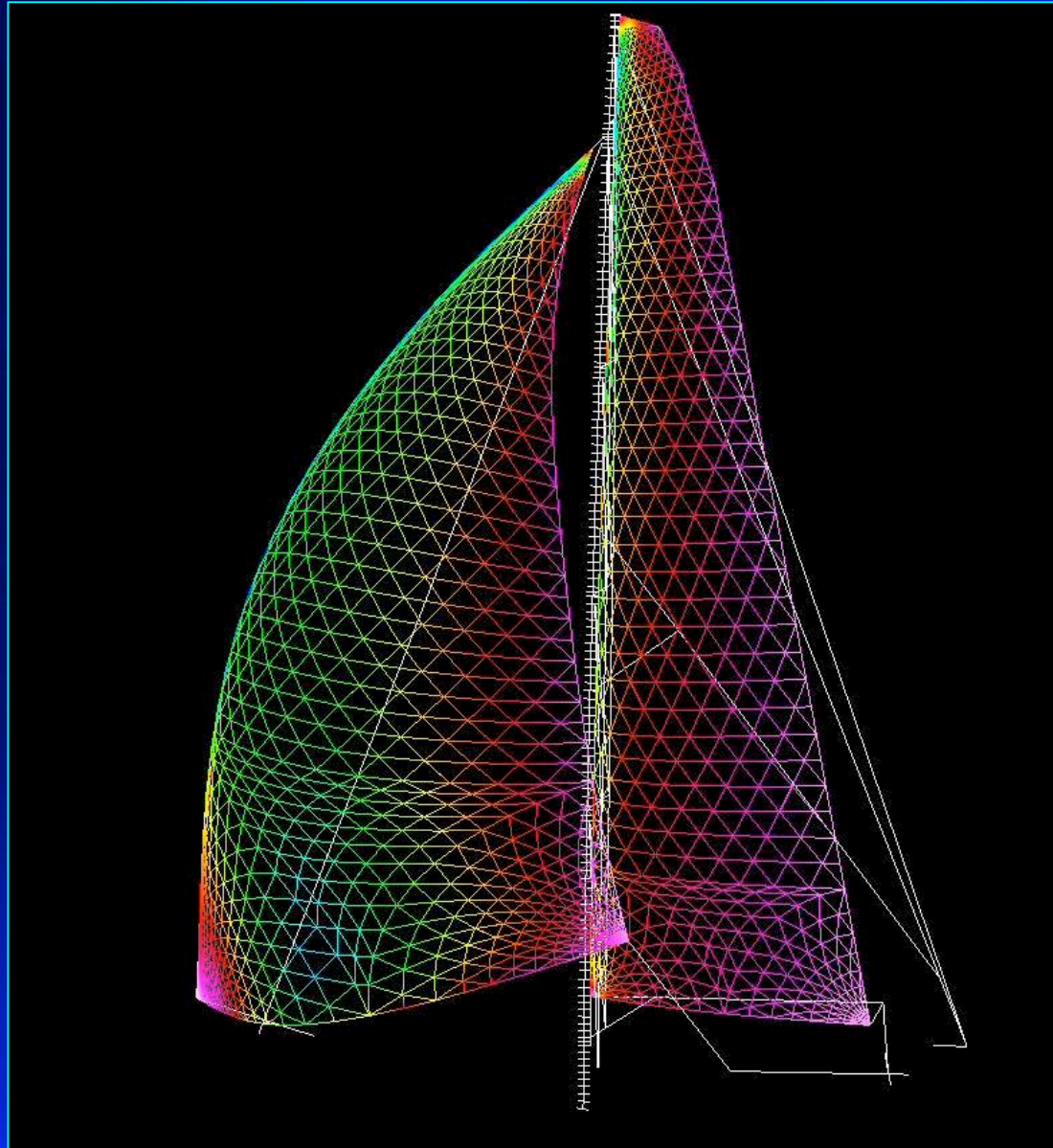
# Example of Spinnaker-Mainsail FSI

- 14 FSI iterations
- Mesh (average values):
  - ~1.360.000 Tetrahedra  
(around sails)
  - ~125.500 Hexahedra  
(far field)
  - ~3500 Pyramids  
(link Tetra/Hexa)
- Stopping test:  
 $\Delta\text{Forces} < 1\%$  for two consecutive couplings on both sails



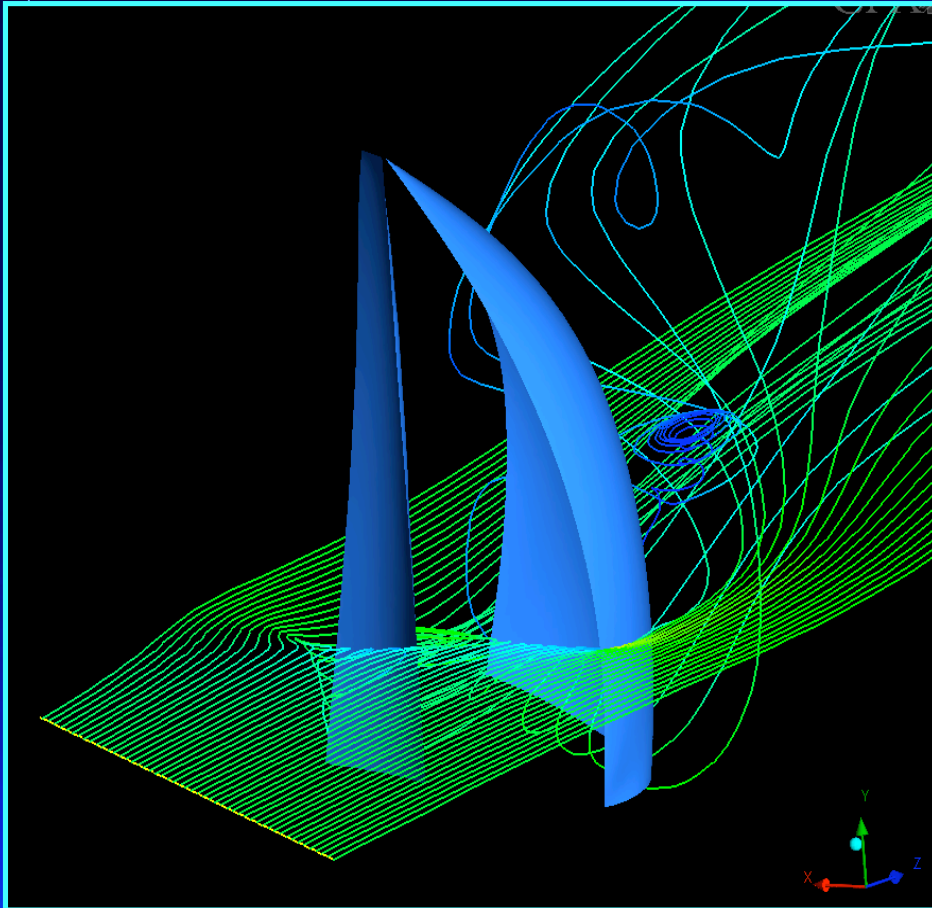


# FE Grid

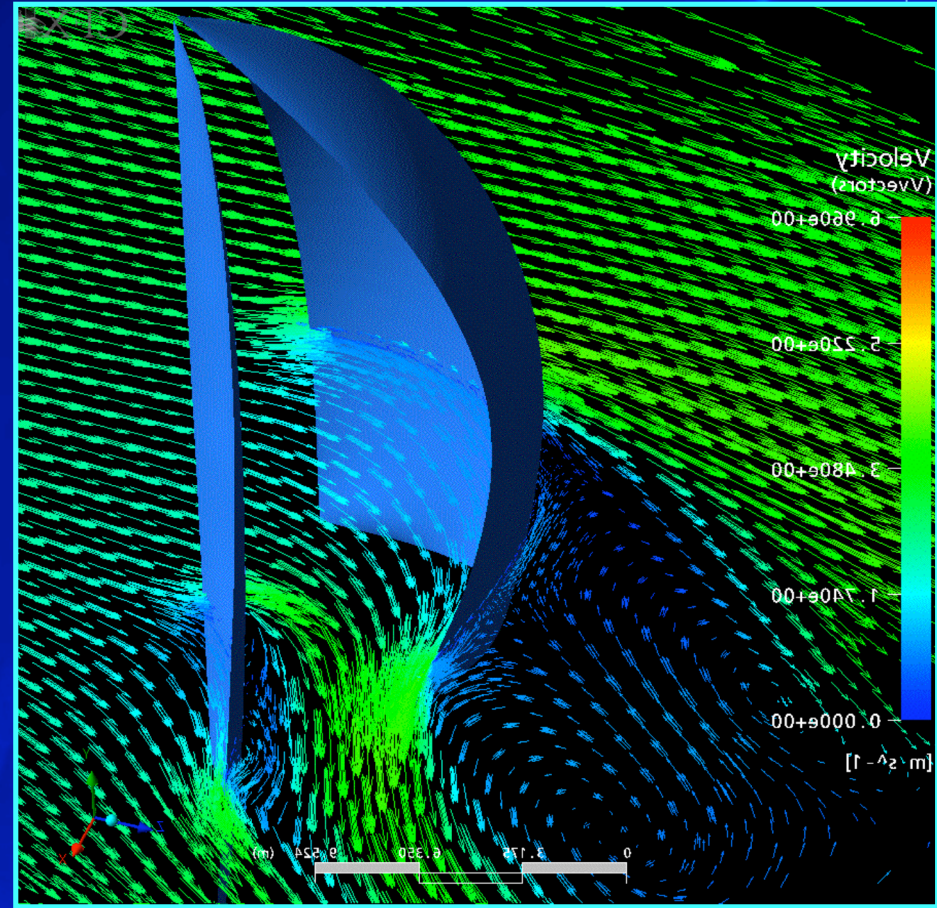


# Flow around spinnaker and mainsail

- Boat speed: 5.540 m/s ( $\sim 15.45$  kts)
- True Wind Angle: 148 Deg
- True Wind Speed at 10m: 5.660 m/s ( $\sim 17.54$  kts)

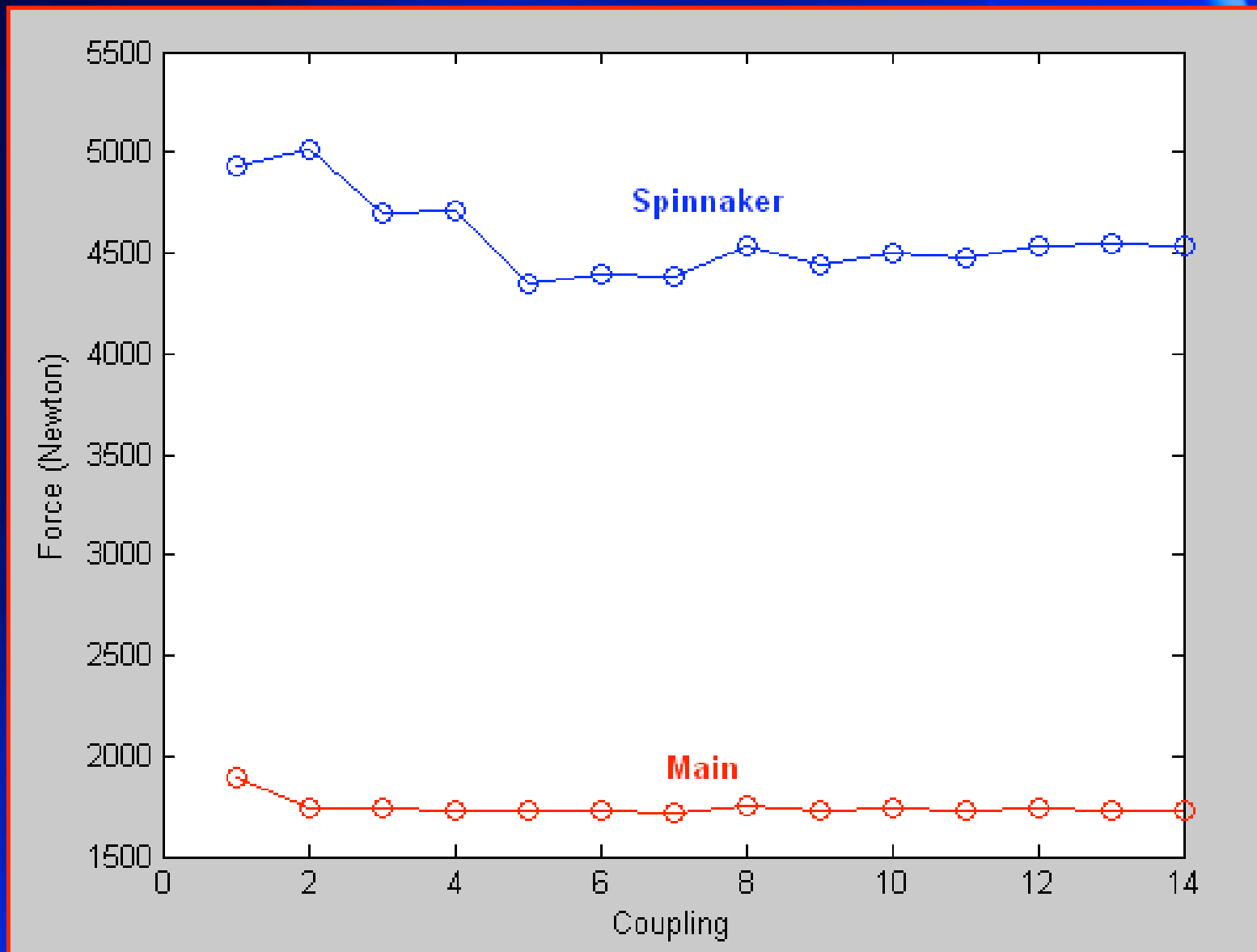


streamlines



velocity and flow separation

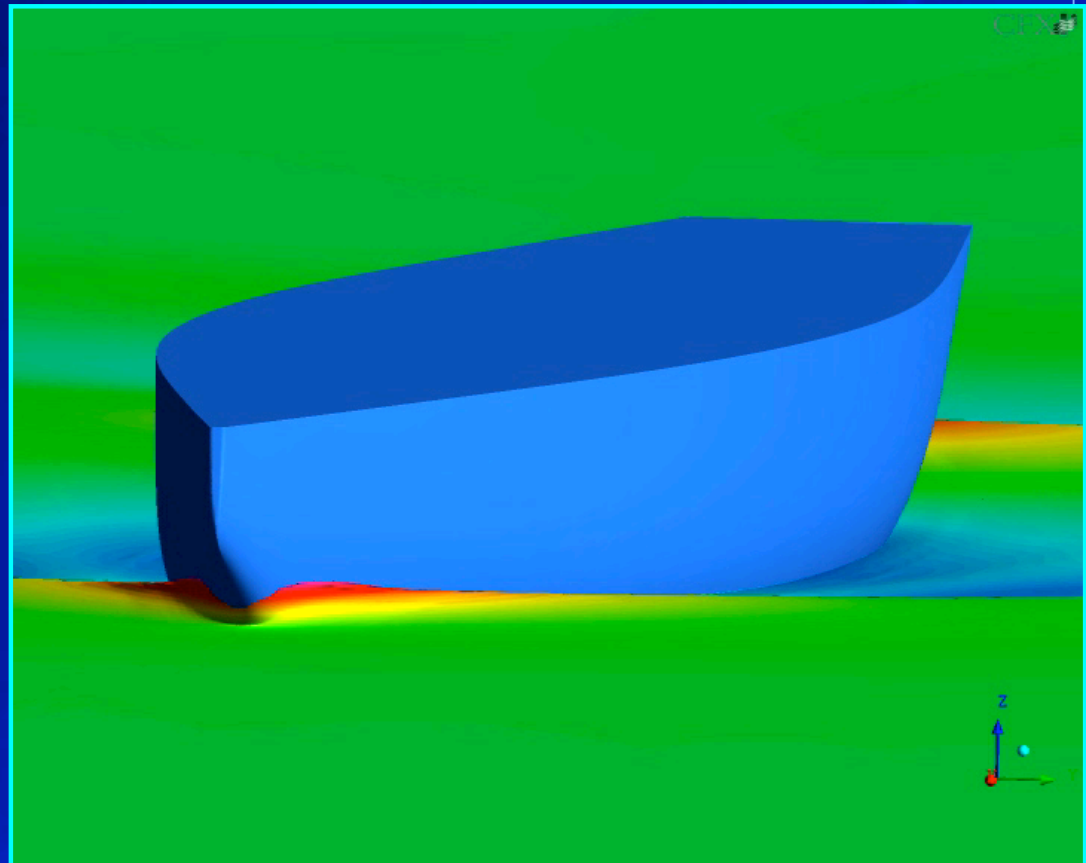
# Convergence history (forces)





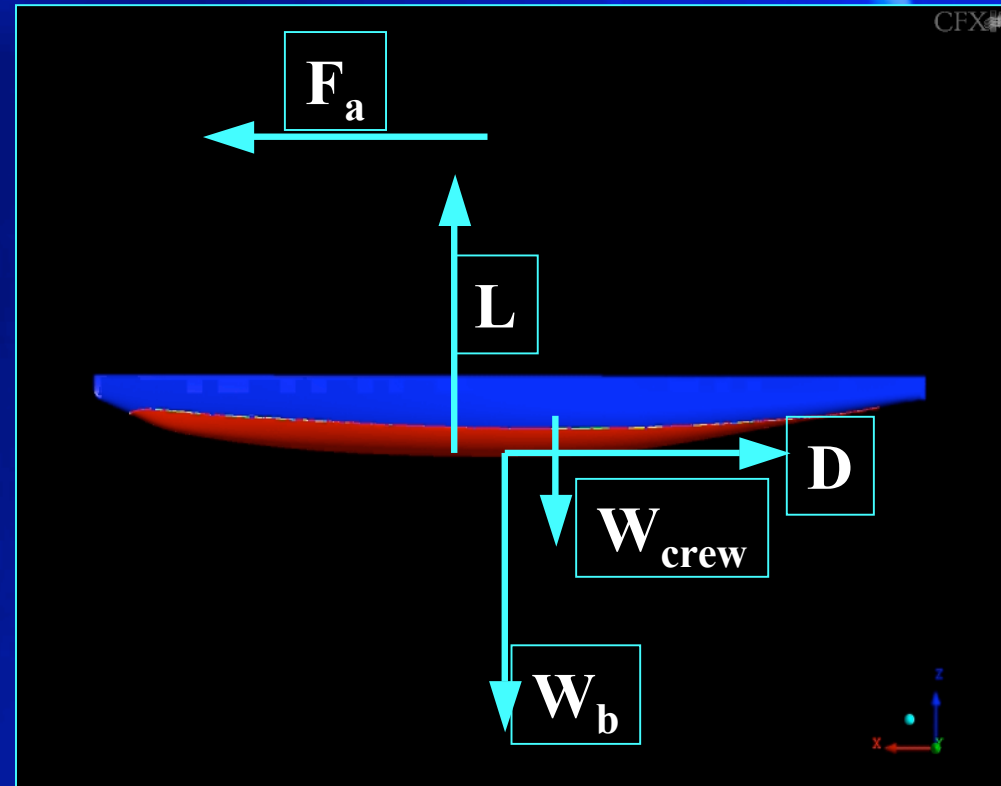
## FSI for hull design

- Coupling between a CFD solver and a rigid body dynamical system for the hull
- Evaluation of forces and sink and trim attitude
- Numerical tool to complement towing tank results on hull performances



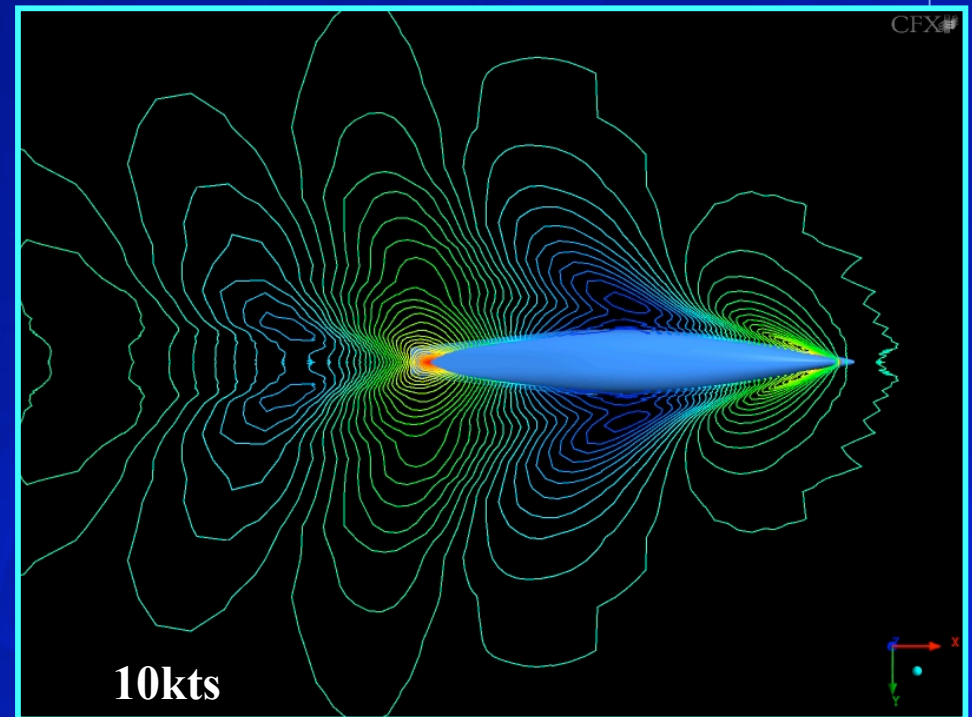
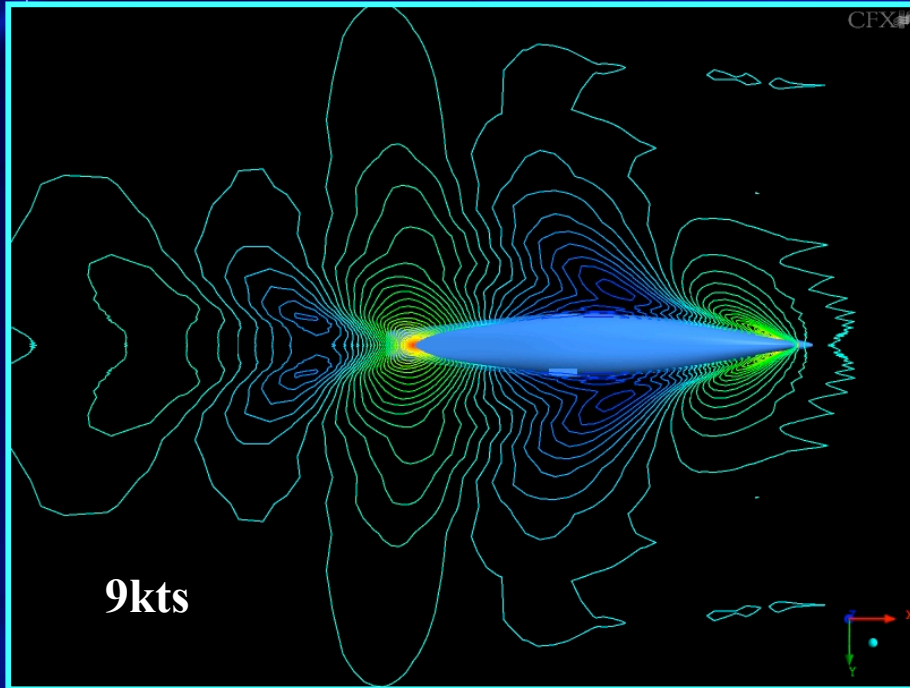
# Rigid body dynamical system

- 2 degrees of freedom:
  - vertical translation (sink)
  - pitching rotation (trim)
- Displacement and external sail moment imposed
- Crew and gear moment imposed
- First order fixed point iteration scheme



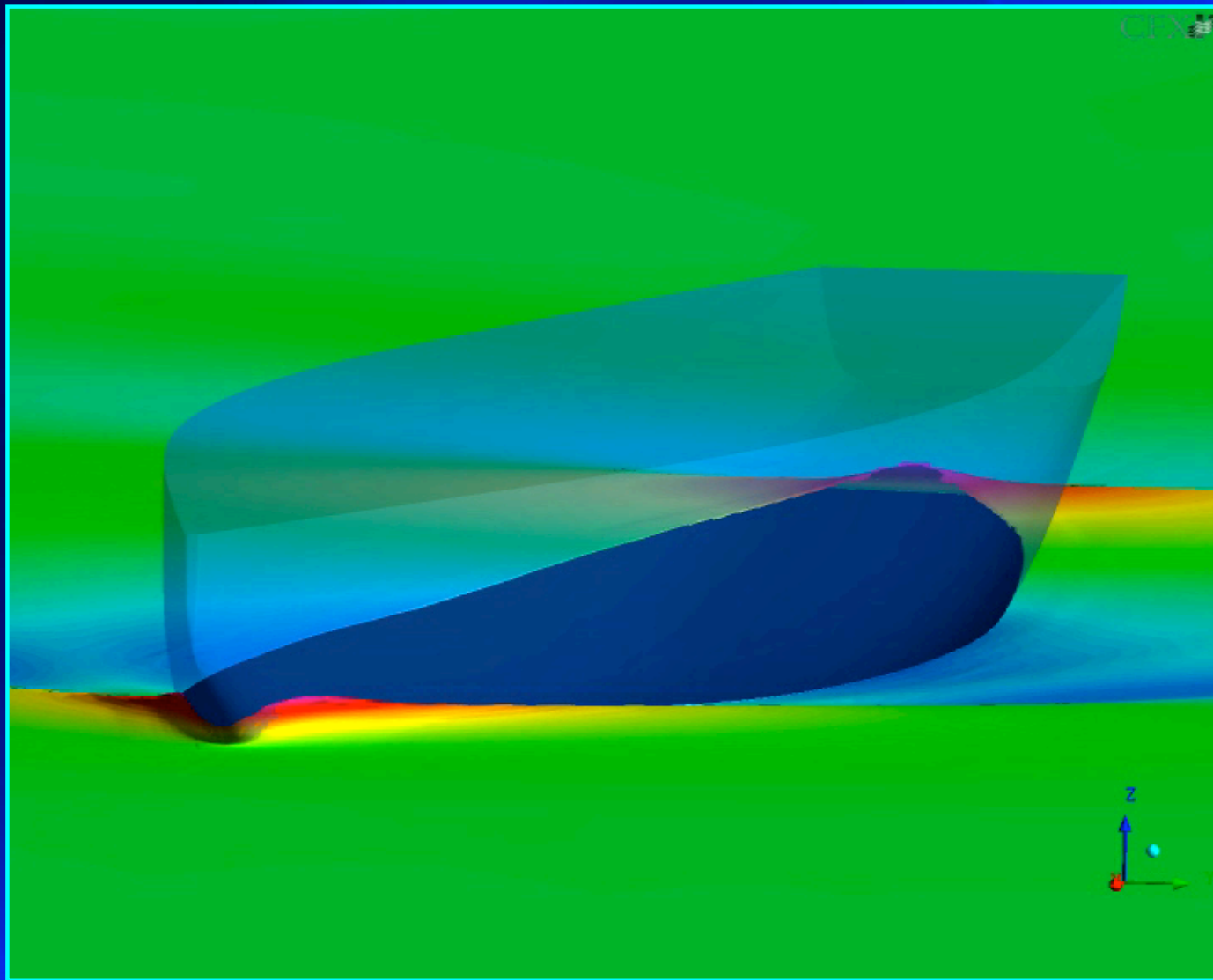
$F_a$ : Sail Thrust  
 $L$ : Hydrodynamic Lift  
 $D$ : Hydrodynamic Drag  
 $W_b$ : Boat weight  
 $W_{crew}$ : Crew weight

# Results - Wave patterns



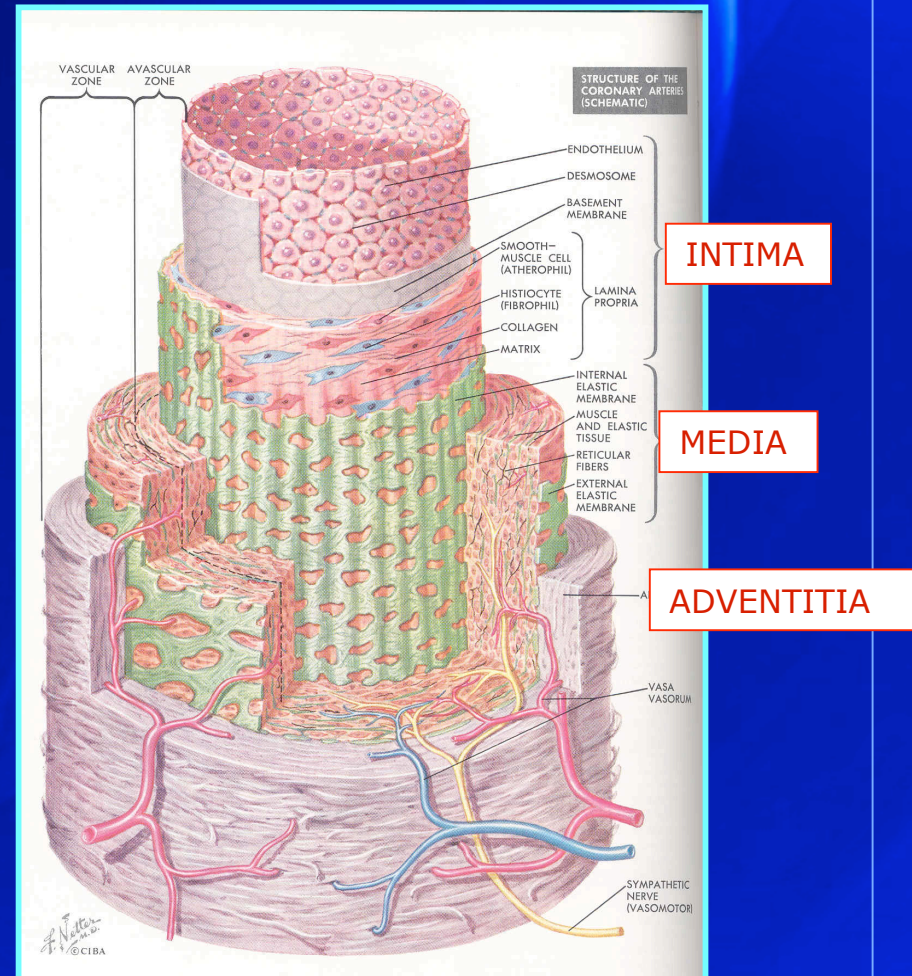


# Hull dynamics

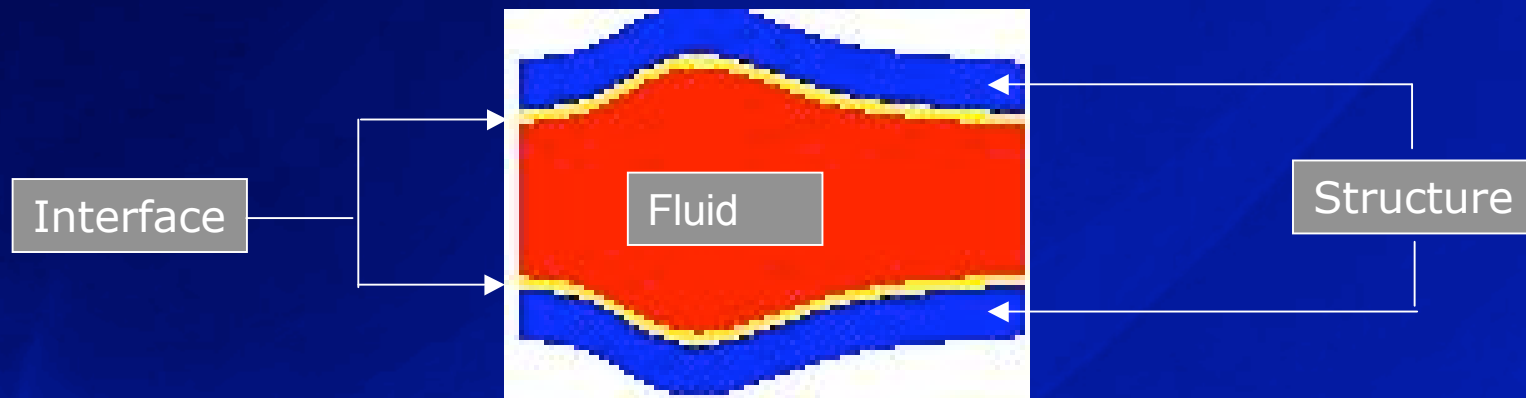


**Boat Speed = 10 kts**

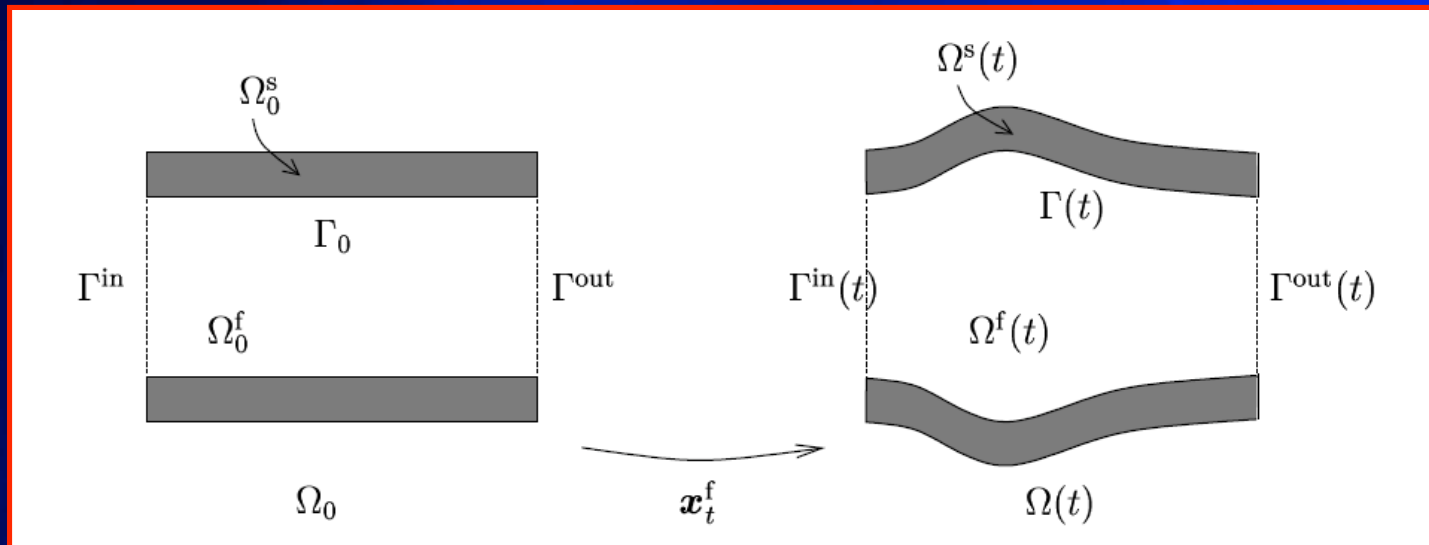
# Application 2: Blood Flow in Large Vessels



# Schematic representation



# ALE Formulation



- Mapping for the solid domain:

$$\forall t, \Omega_0^s \rightarrow \Omega^s(t)$$

$$\mathbf{x}_0 \rightarrow \mathbf{x}^s(\mathbf{x}_0, t) = \mathbf{x}_0 + \mathbf{d}^s(\mathbf{x}_0, t), \mathbf{x}_0 \in \Omega_0^s$$

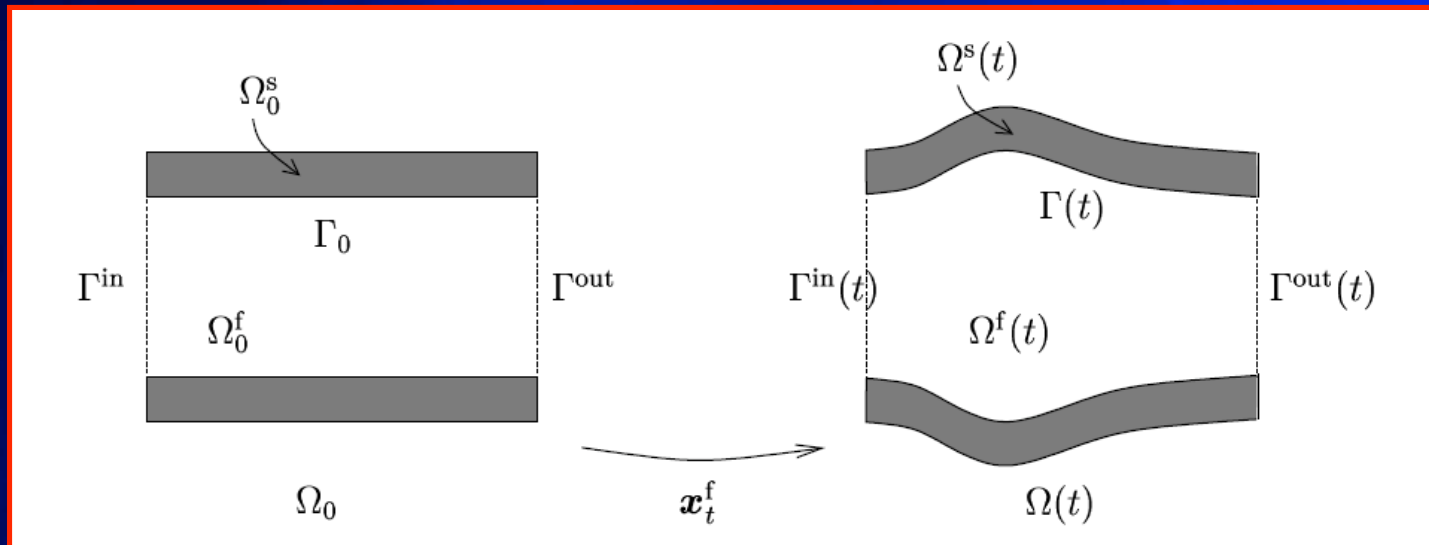
- Mapping for the fluid domain:

$$\forall t, \Omega_0^f \rightarrow \Omega^f(t)$$

$$\mathbf{x}_0 \rightarrow \mathbf{x}^f(\mathbf{x}_0, t) = \mathbf{x}_0 + \mathbf{d}^f(\mathbf{x}_0, t), \mathbf{x}_0 \in \Omega_0^f$$



# ALE Formulation

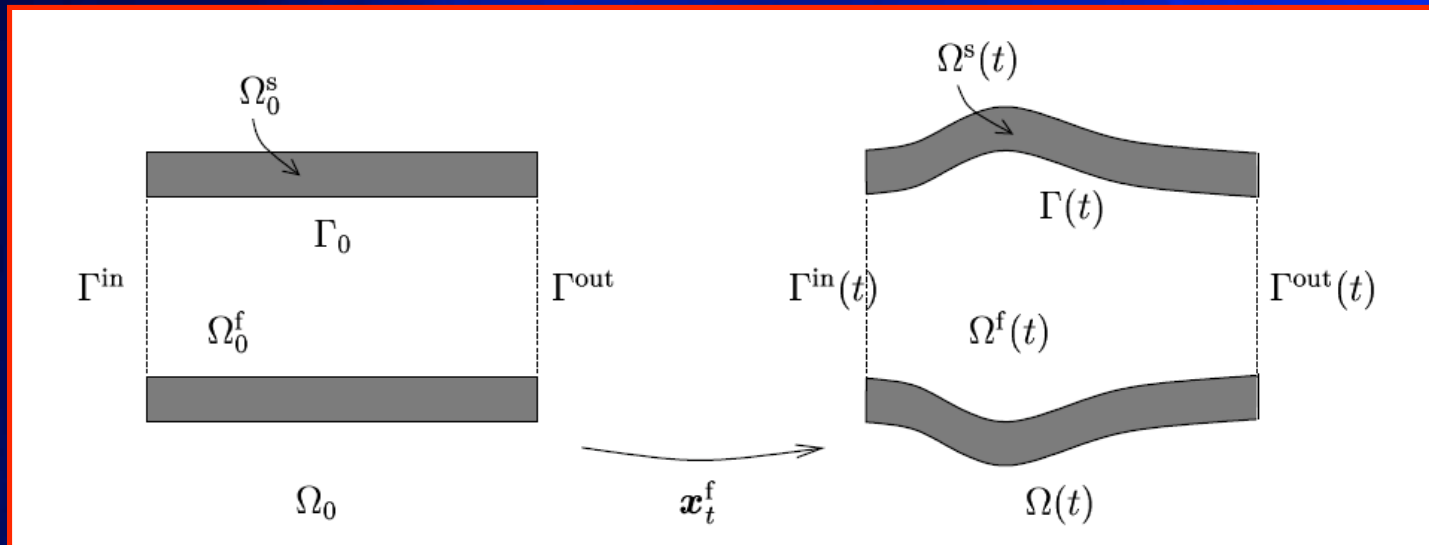


$d^f$  can be defined as an extension of the solid  $d^s|_{\Gamma_0}$

$d^f = \text{Ext}(d^s|_{\Gamma_0})$ , with Ext defined as, e.g.,

$$\begin{aligned} \Delta d^f &= 0 && \text{in } \Omega_0^f \\ d^f &= 0 && \text{on } \partial \Omega_0^f \setminus \Gamma_0 \\ d^f &= d^s && \text{on } \Gamma_0 \end{aligned}$$

# ALE Formulation



- Domain velocity

$$\mathbf{w} = \mathbf{d} \mathbf{x}_t^f / \mathbf{d}t$$

- ALE time derivative

$$\left\{ \frac{\partial \mathbf{u}}{\partial t} \right\}_{|_{\mathbf{x}_0}}(\mathbf{x}, t) = \frac{d\mathbf{u}(\mathbf{x}_t^f(\mathbf{x}_0), t)}{dt} \text{ with } \mathbf{x}_0 = (\mathbf{x}_t^f)^{-1}(\mathbf{x})$$

## Problem Setting

- Fluid( $\mathbf{u}$ ,  $p$ ,  $\mathbf{x}_t^f$ ,  $\mathbf{g}_f$ ,  $\mathbf{f}_f$ )

$$\rho_f(\partial \mathbf{u} / \partial t |_{x_0} + (\mathbf{u} - \mathbf{w}) \cdot \nabla \mathbf{u}) = \nabla \cdot (\boldsymbol{\sigma}_f(\mathbf{u}, p)) + \mathbf{f}_f \quad \text{in } \Omega^f(t)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega^f(t)$$

$$\boldsymbol{\sigma}_f(\mathbf{u}, p) \cdot \mathbf{n}_f = \mathbf{g}_f \quad \text{on } \Gamma^{\text{in}}(t) \cup \Gamma^{\text{out}}(t)$$

- Solid( $\mathbf{d}^s$ ,  $\mathbf{g}_s$ ,  $\mathbf{f}_s$ )

$$\rho_s \partial^2 \mathbf{d}^s / \partial t^2 - \nabla |_{x_0} \cdot (\boldsymbol{\sigma}_s(\mathbf{d}^s)) = \mathbf{f}_s \quad \text{in } \Omega^s_0$$

$$\boldsymbol{\sigma}_s(\mathbf{d}^s) \cdot \mathbf{n}_s = \mathbf{g}_s \quad \text{on } \partial \Omega^s_0 \setminus \Gamma_0$$

- Matching conditions

Let  $\lambda(t) = \lambda$  be an interface variable corresponding to  $\mathbf{d}^s$  on  $\Gamma_0$

$$\mathbf{x}_t^f = \mathbf{x}_0 + \lambda$$

$$\mathbf{u} \circ \mathbf{x}_t^f = \partial \lambda / \partial t$$

$$(\boldsymbol{\sigma}_f(\mathbf{u}, p) \cdot \mathbf{n}_f) \circ \mathbf{x}_t^f = - \boldsymbol{\sigma}_s(\mathbf{d}^s) \cdot \mathbf{n}_s$$

## WARNING:

For time-discretization, geometric conservation laws (GCL) can be a concern for stability. This is a general issue for evolution equations in changing domains.

(see D.Boffi's talk)

Donea,  
Hughes,  
Farhat,  
Nobile and Formaggia,  
Boffi and Gastaldi,  
...



## Strongly vs weakly coupled methods

- Density of structure  $\sim$  density of fluid makes implicit scheme ideal as they guarantee energy conservation (strong coupling: matching conditions satisfied exactly at each time-step)
- Numerical instability observed (and even proven theoretically) for weakly (or loosely) coupled schemes

- Strongly coupled
  - no numerical instabilities
  - high computational costs
- Weakly coupled
  - efficiency and simplicity of implementation
  - unstable when there is an important “added-mass” effect ( $\rho_s \simeq \rho_f$ ) as in blood flows

The fluid acts as an “added-mass” on the structure  
(H.Morand and R.Ohayon, 1995)

## Example: a simple linear fluid-structure problem

**Fluid model (linear incompressible inviscid model:**  
→ potential pressure field)

$$\begin{cases} \Delta p = 0 & \text{in } \Omega_f \\ p = p_{in}(t) & \text{on } \Gamma^{inflow} \\ p = 0 & \text{on } \Gamma^{out} \end{cases}$$

**Structure model (linear elasticity, small thickness cylinder,**  
assumption of membrane deformation: → generalized string model)

$$\rho_s h_s \frac{\partial^2 d}{\partial t^2} - kG h_s \frac{\partial^2 d}{\partial x^2} + \frac{E h_s}{1 - \nu^2} \frac{d}{R_0^2} = \vec{\sigma}_s$$

Physical parameters:  $\rho_s = 1.1$ ,  $h_s = 0.1$ , Poisson  $\nu = 0.5$

Young modulus  $E = 7.5 \cdot 10^5 \text{ dynes/cm}^2$

Shear modulus  $kG = 2.5 \cdot 10^5 \text{ dynes/cm}^2$

Vessel 6 cm long and 1cm wide

## Coupling conditions:

### Continuity of velocity

$$\frac{\partial p}{\partial n} = -\rho_f \frac{\partial^2 d}{\partial t^2}$$

( $\rho_f = 1g/cm^3$  blood density)

### Continuity of stress

$$p = -\vec{\sigma}_s$$

Physical parameters:  $\rho_s = 1.1$ ,  $h_s = 0.1$ , Poisson  $\nu = 0.5$

Young modulus  $E = 7.5 \cdot 10^5 \text{ dynes/cm}^2$

Shear modulus  $kG = 2.5 \cdot 10^5 \text{ dynes/cm}^2$

## Analysis

Explicit time-marching schemes:

The scheme is unconditionally unstable if

$$\frac{\rho s h s}{\rho_f \mu_{max}} < 1$$

with  $\mu_{max}$  maximal eigenvalue of the “added-mass” operator  $\mathcal{M}_A$  (the more  $\Omega_F$  becomes a slender geometry, i.e. for fixed R, the length L increases, or for fixed L, the radius R decreases, the larger  $\mu_{max}$  becomes)  $\mathcal{M}_A$  is the Neumann-to-Dirichlet map.

Implicit time-marching schemes (Implicit Euler):

A Dirichlet-Neumann scheme = fluid solve + structure solve  
converges iff the relaxation parameter  $\omega$  satisfies

$$0 < \omega < \frac{2(\rho s h s + a \delta t^2)}{\rho s h s + \rho_f \mu_{max} + a \delta t^2}$$

(Causin, Gerbeau, Nobile (2004))



## FSI algorithms, I

- Monolithic (direct) method: solve simultaneously the fluid and the structure problems in a unique solver



Strongly coupled by construction

- Partitioned procedure: the fluid and the structure are solved with two different codes (at any rate, separately)



strongly coupled:  
sub-iterations at each time  
step until convergence



most often weakly coupled:  
a single fluid-structure  
solve at each time step , or  
a few (inexact solution)

See Hermann Matthies' lecture on strongly coupled approaches

## FSI algorithms, II

1. Fixed point, Gauss-Seidel or Schwarz multiplicative
2. Newton based methods: requires the evaluation of the Jacobian associated to fluid-solid coupled state equations
  - 2a. Exact Newton
  - 2b. Block, Quasi, or Inexact-Jacobian Newton
3. Fractional step schemes: differential and algebraic
4. Schur-based domain decomposition

Several analogies exist among these strategies

# FSI algorithms, III: fixed point and Newton

- Fixed point are common practice, with several variants: steepest descent, Aitken acceleration, transpiration conditions, to avoid computation of the fluid matrix at each iteration.

Methods slow (in general) or even non-convergent (unless properly relaxed and/or accelerated), depending upon the physical characteristics of the two media.

- Remedy: use Newton method, however it requires the Jacobian evaluation (for FS system). In particular, the cross-jacobian expresses the sensitivity of the fluid state to solid motion.
- Cross Jacobian can be evaluated inexactly: FD approximation of derivatives, or by replacing the tangent operator by a simpler one. To recover convergence, acceleration techniques based on Krylov methods have been proposed.

# FSI algorithms, IV: Fractional steps, differential and algebraic (weakly coupled, variable degree)

1. Projection semi-implicit scheme
2. Algebraic fractional step scheme

They couple implicitly the pressure stress to the structure

→ implicit coupling of the added mass term

→ good stability properties

The remaining terms of the fluid equations are explicitly coupled



## Projection and Fractional step: common steps

- Step 0: extrapolation of FS interface

$$\bar{\eta}^{n+1} = \eta^n + \delta t \left( \frac{3}{2} \dot{\eta}^n - \frac{1}{2} \dot{\eta}^{n-1} \right)$$

- Step 1: definition of the new domain (and ALE velocity)

$$\mathbf{w}^{n+1} |_{\Gamma} = \frac{\bar{\eta}^{n+1} - \eta^n}{\delta t} |_{\Gamma} \quad \mathbf{w}^{n+1} = \text{Ext}(\mathbf{w}^{n+1} |_{\Gamma})$$

$$\Omega_f^{n+1} = \Omega_f^n + \delta t \mathbf{w}^{n+1}$$

ALE framework :  $\mathbf{w}^{n+1}$  is the fluid domain velocity at time  $t^{n+1}$

## Projection based *semi-implicit* coupling (case: low Re)

It splits the **differential** operator (Chorin-Temam projection scheme) and then discretizes in time and space

- Step 2: diffusion step (explicit coupling):

$$\rho_f \frac{\tilde{\mathbf{u}}^{n+1} - \mathbf{u}^{n+1}}{\delta t} \Big|_{\hat{x}} - \mu \Delta \tilde{\mathbf{u}}^{n+1} = 0 \quad \text{in } \Omega_f^{n+1}$$
$$\tilde{\mathbf{u}}^{n+1} = \mathbf{w}^{n+1} \quad \text{on } \Gamma^{n+1}$$

- Step 3: projection step (implicit coupling):

- Step 3.1:

$$\rho_f \frac{\mathbf{u}^{n+1} - \tilde{\mathbf{u}}^{n+1}}{\delta t} + \nabla p^{n+1} = 0 \quad \text{in } \Omega_f^{n+1}$$
$$\operatorname{div} \mathbf{u}^{n+1} = 0 \quad \text{in } \Omega_f^{n+1}$$
$$\mathbf{u}^{n+1} \cdot \mathbf{n}^f = \frac{\eta^{n+1} - \eta^n}{\delta t} \cdot \mathbf{n}^f \quad \text{on } \Gamma^{n+1}$$

## Projection based *semi-implicit* coupling (II)

- Step 3.2 (mid-point rule discretization for the structure):

$$\rho_s \frac{\dot{\eta}^{n+1} - \dot{\eta}^n}{\delta t} - \operatorname{div}_{\hat{x}} \left( \frac{\sigma_s^{n+1} + \sigma_s^n}{2} \right) = 0 \quad \text{in } \hat{\Omega}_s$$

$$\frac{\eta^{n+1} - \eta^n}{\delta t} = \frac{\dot{\eta}^{n+1} + \dot{\eta}^n}{2} \quad \text{in } \hat{\Omega}_s$$

$$\sigma_s^{n+1} \cdot \mathbf{n}^s = \sigma_{f, \hat{x}}^{n+1}(\tilde{\mathbf{u}}^{n+1}, p^{n+1}) \cdot \mathbf{n}_{\hat{x}}^f \quad \text{on } \hat{\Gamma}$$

Step 1 and 2 are performed only once per time step

Step 3 is solved by sub-iterating (in a fixed domain) between 3.1 and 3.2, e.g. using fixed-point or Newton iterations.

# Schur-based domain decomposition

## Variational Formulation

Harmonic Extension:

find  $d^{f,t^{n+1}} \in H^1(\Omega^1_0)$  such that

$$\begin{aligned} \int_{\Omega^f_0} \nabla d^{f,t^{n+1}} \cdot \nabla \phi &= 0 \\ d^{f,t^{n+1}} &= \lambda(t^{n+1}) \quad \text{on } \Gamma_0 \end{aligned}$$

for all  $\phi \in H^1_0(\Omega^1_0)^3$  with appropriate boundary conditions on  $\Gamma^{\text{in}} \cup \Gamma^{\text{out}}$

We can then compute the velocity of the fluid domain:

$$w^{f,n+1}|_{\Gamma(t^{n+1})} = 1/\delta t (d^{f,t^{n+1}}|_{\Gamma_0} - d^{f,t^n}|_{\Gamma_0}) \circ (x^{f,t^{n+1}})^{-1}$$



## Variational Formulation

**Fluid:** Find  $(\mathbf{u}^{n+1}, p^{n+1}) \in V^f(t^{n+1}) \times Q^f(t^{n+1})$  such that (Dirichlet data)

$$\mathbf{u}^{n+1}|_{\Gamma(t^{n+1})} = \mathbf{w}^{f,n+1}|_{\Gamma(t^{n+1})}$$

$$\mathbf{u}^{n+1}|_{\Gamma^{\text{in}}(t^{n+1})} = \mathbf{u}_{\text{in}}(t^{n+1})$$

$$\begin{aligned} & 1/\delta t \int_{\Omega^f(t^{n+1})} \rho_f \mathbf{u}^{n+1} \cdot \mathbf{v}^f + \int_{\Omega^f(t^{n+1})} \rho_f [(\mathbf{u}^{n+1} - \mathbf{w}^{f,n+1}) \cdot \nabla \mathbf{u}^{n+1}] \cdot \mathbf{v}^f + \mu \int_{\Omega^f(t^{n+1})} \sigma_f(\mathbf{u}^{n+1}, p^{n+1}) \cdot \nabla \mathbf{v}^f = 1/\delta t \int_{\Omega^f(t^{n+1})} \rho_f \mathbf{u}^n \cdot \mathbf{v}^f \\ & + \int_{\Gamma^{\text{in}}(t^{n+1}) \cup \Gamma^{\text{out}}(t^{n+1})} \mathbf{g}^f \cdot \mathbf{v}^f \end{aligned}$$

$$\int_{\Omega^f(t^{n+1})} q^f \nabla \cdot \mathbf{u}^{n+1} = 0$$

$\forall (\mathbf{v}^f, q^f) \in V_0^f(t^{n+1}) \times Q^f(t^{n+1})$  with

$$V^f(t) = \{\mathbf{v}^f \mid \mathbf{v}^f \circ \mathbf{x}_t^f \in H^1(\Omega_0^f)^3\}$$

$$V_0^f(t) = \{\mathbf{v}^f \in V^f(t) \mid \mathbf{v}^f \circ \mathbf{x}_t^f = 0 \text{ on } \Gamma_0 \cup \Gamma^{\text{in}}\}$$

$$Q^f(t) = \{q^f \mid q^f \circ \mathbf{x}_t^f \in L^2(\Omega_0^f)\}$$

## Variational Formulation

Structure (Neumann data):

$$\begin{aligned} & 2/\delta t^2 \int_{\Omega^s_0} \rho_s \mathbf{d}^{s,n+1} \cdot \mathbf{v}^s - 2/\delta t^2 \int_{\Omega^s_0} \rho_s (\mathbf{d}^{s,n} + \delta t \mathbf{w}^{s,n}) \cdot \mathbf{v}^s \\ & + \int_{\Omega^s_0} \boldsymbol{\sigma}_s(\mathbf{d}^{s,n+1}) \cdot \nabla \mathbf{v}^s = \int_{\partial \Omega^s_0 \setminus \Gamma_0} \mathbf{g}_s \cdot \mathbf{v}^s \\ & + \int_{\Gamma_0} \boldsymbol{\sigma}_f(\mathbf{u}^{n+1}, p^{n+1}) \cdot \mathbf{x}^f_{t^{n+1}} \end{aligned}$$

$\forall \mathbf{v}^s \in V^s$  such that  $\mathbf{v}^s|_{\Gamma_0} = 0$ , and  $V^s = H^1(\Omega^s_0)^3$

$$\mathbf{w}^{s,n+1} = 2/\delta t(\mathbf{d}^{s,n+1} - \mathbf{d}^{s,n}) - \mathbf{w}^{s,n}$$

$$\mathbf{d}^{s,n+1} = \boldsymbol{\lambda}(t^{n+1}) \text{ on } \Gamma_0$$

## Interface Operators

Fluid operator  $S_f$ : (Dirichlet-to-Neumann)

$$S_f(\lambda) = \sigma_f := (\sigma_f(u,p) \cdot n_f) \circ x_t^f \text{ on } \Gamma_0$$

where  $(u,p)$  is the solution of the Navier-Stokes problem

Structure operator  $S_s$ : (Dirichlet-to-Neumann)

$$S_s(\lambda) = \sigma_s := (\sigma_s(d^s) \cdot n_s) \text{ on } \Gamma_0$$

where  $d^s$  is the solution of the structure problem

We can also define the associated inverse and derived operators

$$S_f^{-1}, S_s^{-1}, S_f^{\prime -1}, S_s^{\prime -1}, \dots$$

## Interface Problem

Fixed Point Formulation:

$$S_s^{-1}(-S_f(\lambda)) = \lambda$$

relaxed fixed point iterations:

$$\Lambda^k = S_s^{-1}(-S_f(\lambda^k))$$

$$\lambda^{k+1} = \lambda^k + \alpha^k(\Lambda^k - \lambda^k)$$

With:

$\alpha^k$  is a relaxation parameter (constant, Aitken's method, ...)

$J_\Phi$  is the Jacobian of  $S_s^{-1}(-S_f(\lambda))$

$$J_\Phi(\lambda^k) = - [S_s'(\Lambda^k)]^{-1} \cdot S_f'(\lambda^k)$$

Rootfinding Formulation:

$$\Phi(\lambda) := S_s^{-1}(-S_f(\lambda)) - \lambda = 0$$

Newton algorithm :

$$\Lambda^k = S_s^{-1}(-S_f(\lambda_k))$$

$$J_\Phi(\lambda^k)\mu^k = -(\Lambda^k - \lambda^k)$$

$$\lambda^{k+1} = \lambda^k + \alpha^k\mu^k$$



## Interface Problem

Steklov-Poincaré Formulation:

$$S_s(\lambda) + S_f(\lambda) = 0$$

Non-Linear Richardson:

$$\sigma^k = - (S_s(\lambda^k) + S_f(\lambda^k))$$

$$\mu^k = P^{-1}\sigma^k$$

$$\lambda^{k+1} = \lambda^k + \alpha^k \mu^k$$

How do we define the preconditioner  $P^{-1}$  ?

## Interface Problem

Domain Decomposition preconditioners:

$$P^{-1} = \alpha_f S_f^{-1} + \alpha_s S_s^{-1}$$

$$\alpha_f + \alpha_s = 1$$

Problem: the operators  $S_f$  and  $S_s$  are non-linear

Idea: replace  $S_f$  and  $S_s$  by  $S_f'$  and  $S_s'$

## Interface Problem: Preconditioning the Richardson Algorithm

Generalized Aitken:

Let  $\alpha_{f/s}^k = \alpha^k \alpha_{f/s}^k$  and

$$\mu_{f/s}^k = -(S_f'(\lambda^k) + S_s'(\lambda^k))(\alpha_f^k S_f'(\lambda^k)^{-1} + \alpha_s^k S_s'(\lambda^k)^{-1}) \sigma^k$$

We want to minimize (w.r.t  $\alpha_f$  and  $\alpha_s$ )

$$|| (\lambda^k - \lambda^{k-1} + \alpha_f(\mu_f^k - \mu_f^{k-1}) + \alpha_s(\mu_s^k - \mu_s^{k-1})) ||$$

Which corresponds to solving the linear system

$$A^T A(\alpha_f^k, \alpha_s^k)^T = -A^T(\lambda^k - \lambda^{k-1})$$

where  $A$  is the two column matrix

$$A = ((\mu_f^k - \mu_f^{k-1}); (\mu_s^k - \mu_s^{k-1}))$$

## Reconsidering the previous Example:

A vessel 6 cm long and 1cm wide

Fluid model (linear incompressible inviscid model  
→ potential pressure field)

$$\begin{cases} \Delta p = 0 & \text{in } \Omega_f \\ p = p_{in}(t) & \text{on } \Gamma^{inflow} \\ p = 0 & \text{on } \Gamma^{out} \end{cases}$$

Structure model (string model)

$$\rho_s h_s \frac{\partial^2 d}{\partial t^2} - kG h_s \frac{\partial^2 d}{\partial x^2} + \frac{E h_s}{1 - \nu^2} \frac{d}{R_0^2} = \vec{\sigma}_s$$

Physical parameters:  $\rho_s = 1.1$ ,  $h_s = 0.1$ , Poisson  $\nu = 0.5$

Young modulus  $E = 7.5 \cdot 10^5 \text{ dynes/cm}^2$

Shear modulus  $kG = 2.5 \cdot 10^5 \text{ dynes/cm}^2$



## DD with Flexible Neumann-Neumann preconditioner

On the same problem (both string and fluid acting as preconditioners)

For fixed  $h = 0.15$

(  $\omega_f$  average of  $\omega_f^k$ ;  $\omega_s$  average of  $\omega_s^k$  )

$\delta t$	$\omega_f$	$\omega_s$	iterations
1.e-4	3.e-3	3.e-2	33
1.e-5	2.e-3	3.8e-2	30
1.e-6	2.8e-3	3.9e-2	27

For fixed  $\delta t = 1.e - 4$

$h$	$\omega_f$	$\omega_s$	iterations
0.3	6.e-3	8.e-2	33
0.15	3.e-3	3.e-2	33
0.1	3.3e-3	2.e-2	37

## The 3D case

### 3D Geometry:

Straight cilinder of radius  $R=0.25\text{cm}$  and length  $L=5\text{cm}$

Structure with thickness of  $0.05\text{cm}$

### Physical characteristics:

*Fluid:* viscosity  $\mu=0.03$  poise, density  $\rho_f=1$  g/cm<sup>3</sup>

*Solid:* density  $\rho_s=1.2$  g/cm<sup>3</sup>, Young modulus  $E = 3.e6$  dynes/cm<sup>3</sup>,

Poisson ratio  $\nu = 0.3$

### Space discretization

*Fluid:* Navier-Stokes equations,  $\mathbb{P}_1$  -bubble/  $\mathbb{P}_1$

*Solid:* Linear Saint Venant-Kirckhoff,  $\mathbb{P}_1$ , no time derivative

### Time discretization

Time interval  $[0,0.02\text{s}]$  and  $\Delta t = 1.e-3, 5.e-4, 1e-4$ .

At  $t=0$  the system is at rest. The structure is clamped at inlet and outlet. Pressure of  $1.3e4$  dynes/cm<sup>2</sup> imposed at the inlet for  $3.e-3\text{s}$