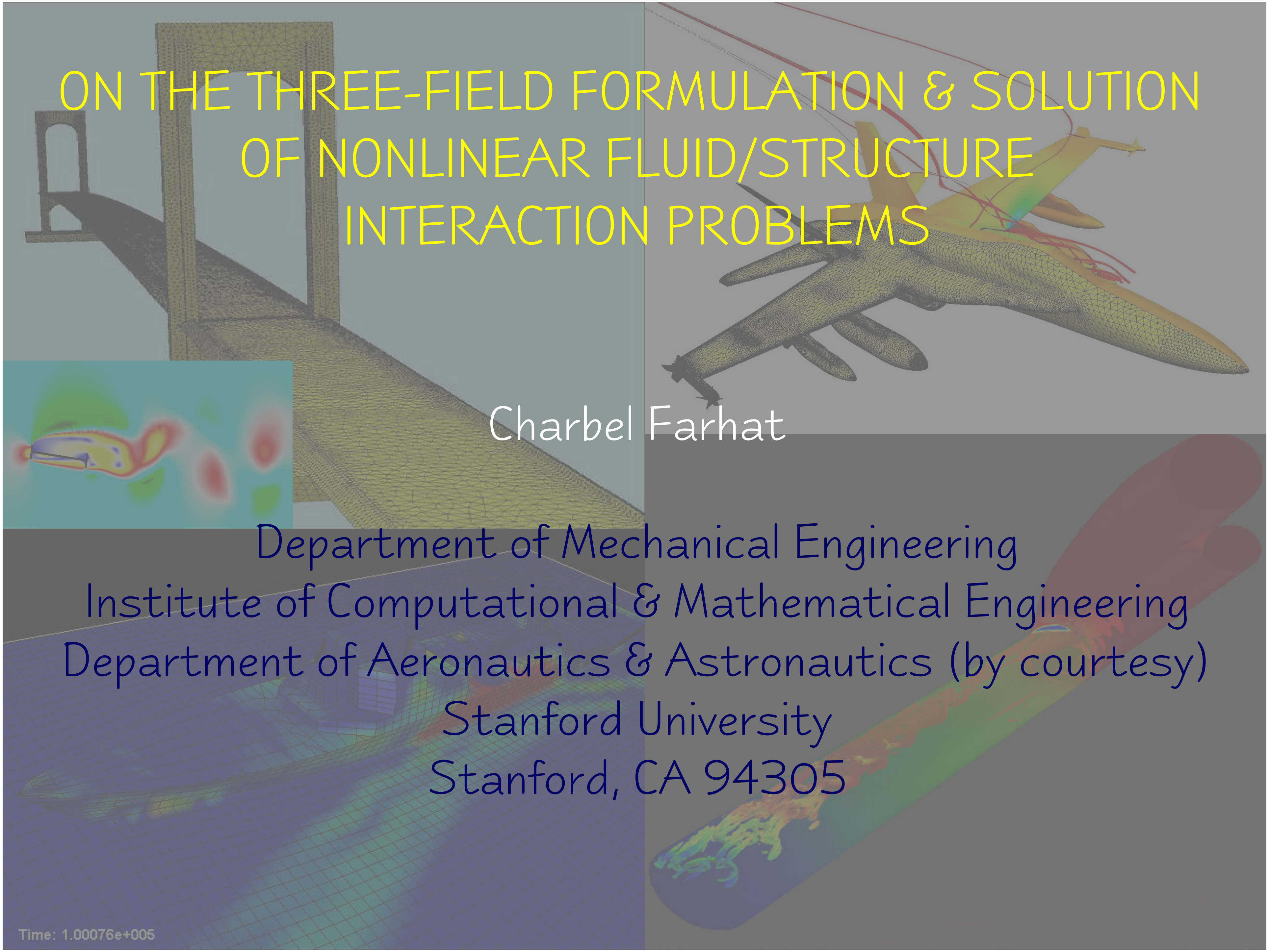

ON THE THREE-FIELD FORMULATION & SOLUTION OF NONLINEAR FLUID/STRUCTURE INTERACTION PROBLEMS

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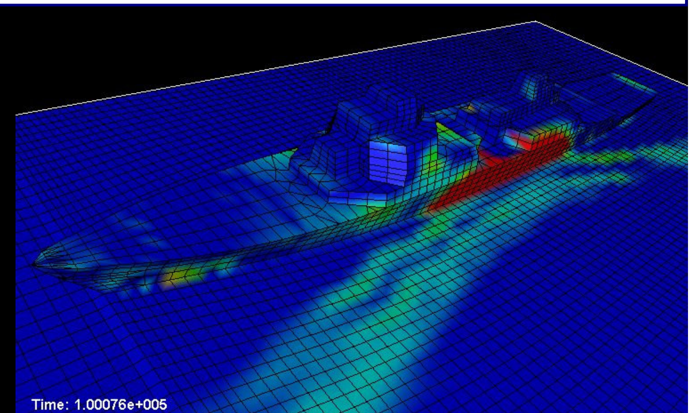
Stanford, CA 94305

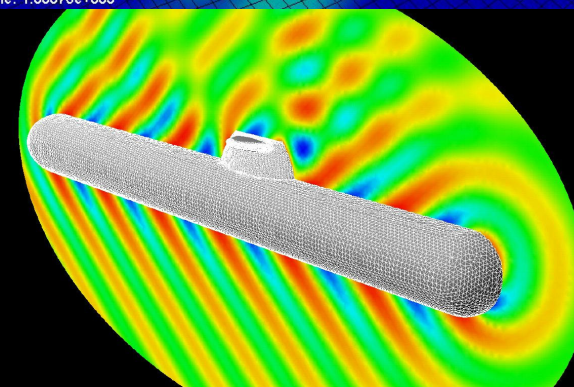
FLUID-STRUCTURE INTERACTION

STANFORD MECHANICAL ENGINEERING

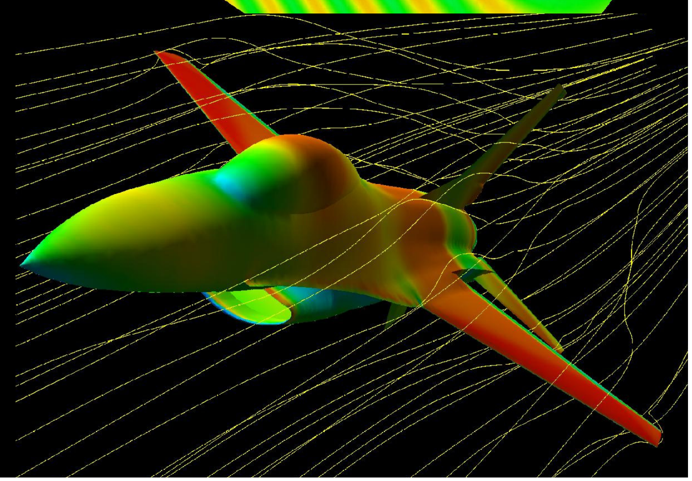
- Class I
  - short duration
  - limited fluid displacements
  - shock, impact



- Class II
  - long duration
  - limited fluid displacements
  - elastoacoustics



- Class III
  - large relative motion
  - process dominated by the flow
  - aeroelasticity



- Time-domain
  - Fluid subsystem
    * Navier-Stokes (laminar/turbulent)
      * Euler
        * Linearized Euler
          * Linearized Euler + small movements
  - Structure subsystem
    * Nonlinear
      * Linear

➡ Class I, Class II and Class III applications

FLUID-STRUCTURE INTERACTION

STANFORD MECHANICAL ENGINEERING

- Frequency-domain
  - Fluid subsystem
    * Linearized Euler + small movements
  - Structure subsystem
    * Linear vibrations (elastodynamics)

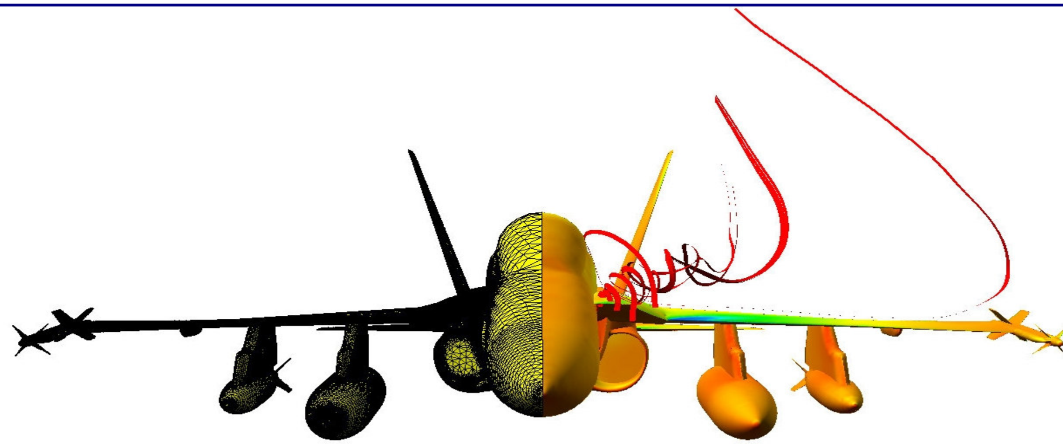
➡ mainly Class II applications

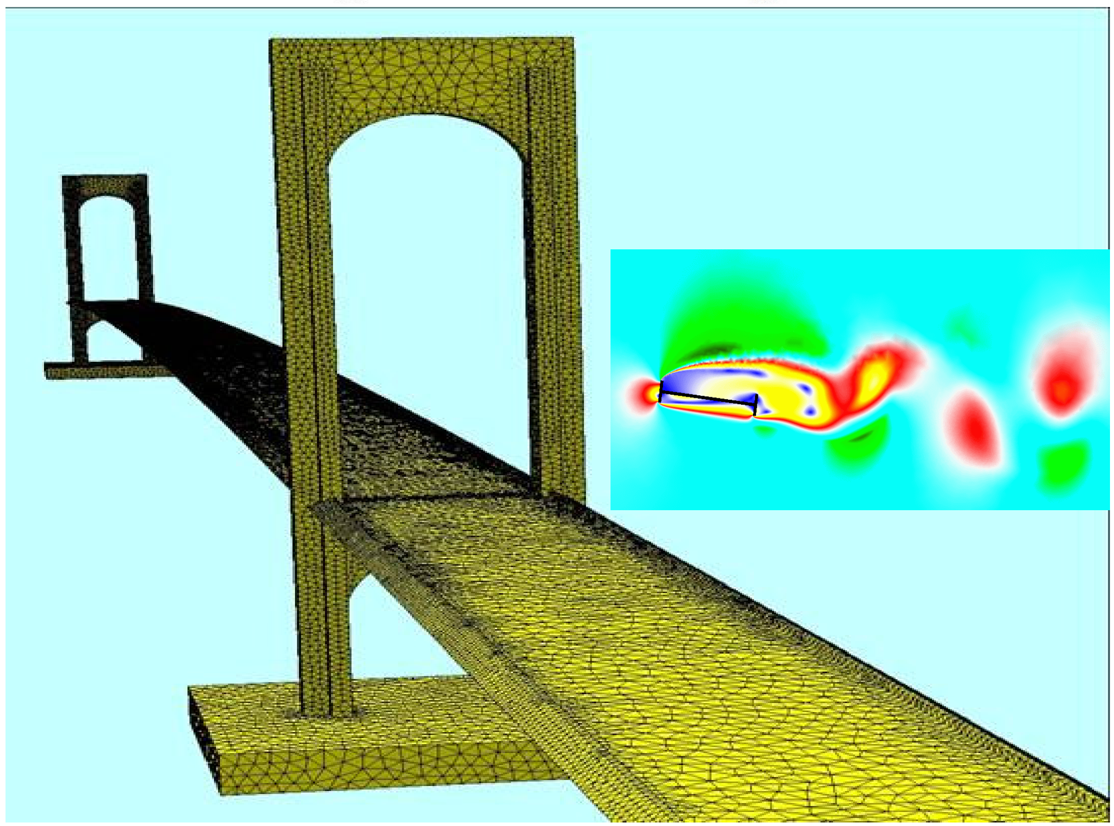
FOCUS APPROACH & APPLICATIONS

- Time-domain
  - Fluid subsystem
    - * Navier-Stokes (laminar/turbulent)
    - * Euler
  - Structure subsystem
    - * Nonlinear
    - * Linear

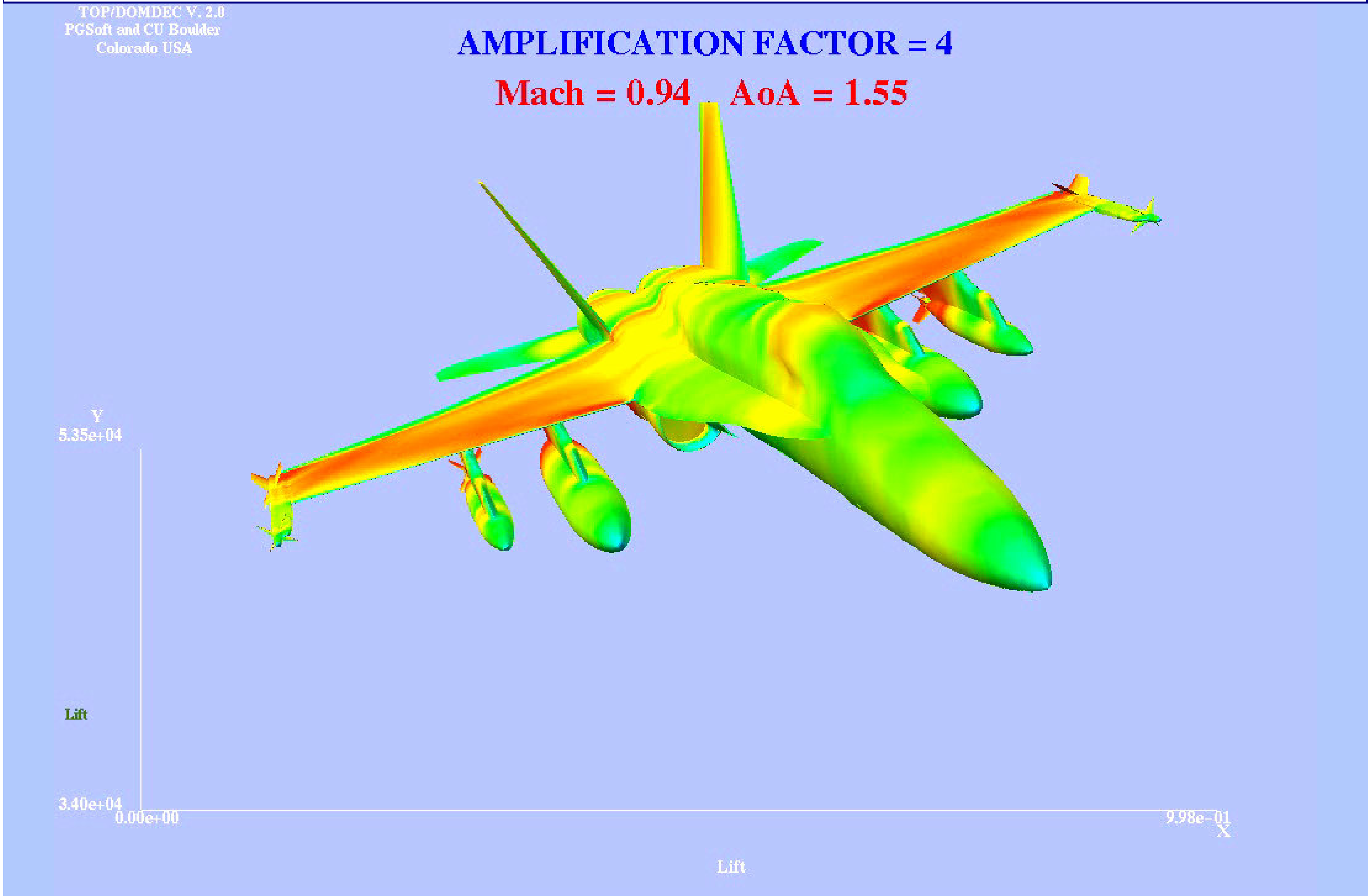
→ mainly Class III applications

EXAMPLES





NONLINEAR AEROELASTICITY



CONTINUOUS INTERFACE MOTION

- Regridding techniques
- Transpiration methods
- Arbitrary Lagrangian-Eulerian methods (dynamic meshes, moving grids, ...)
- Level set methods

FOUR-FIELD FORMULATION

ALE Fluid Flow Formulation

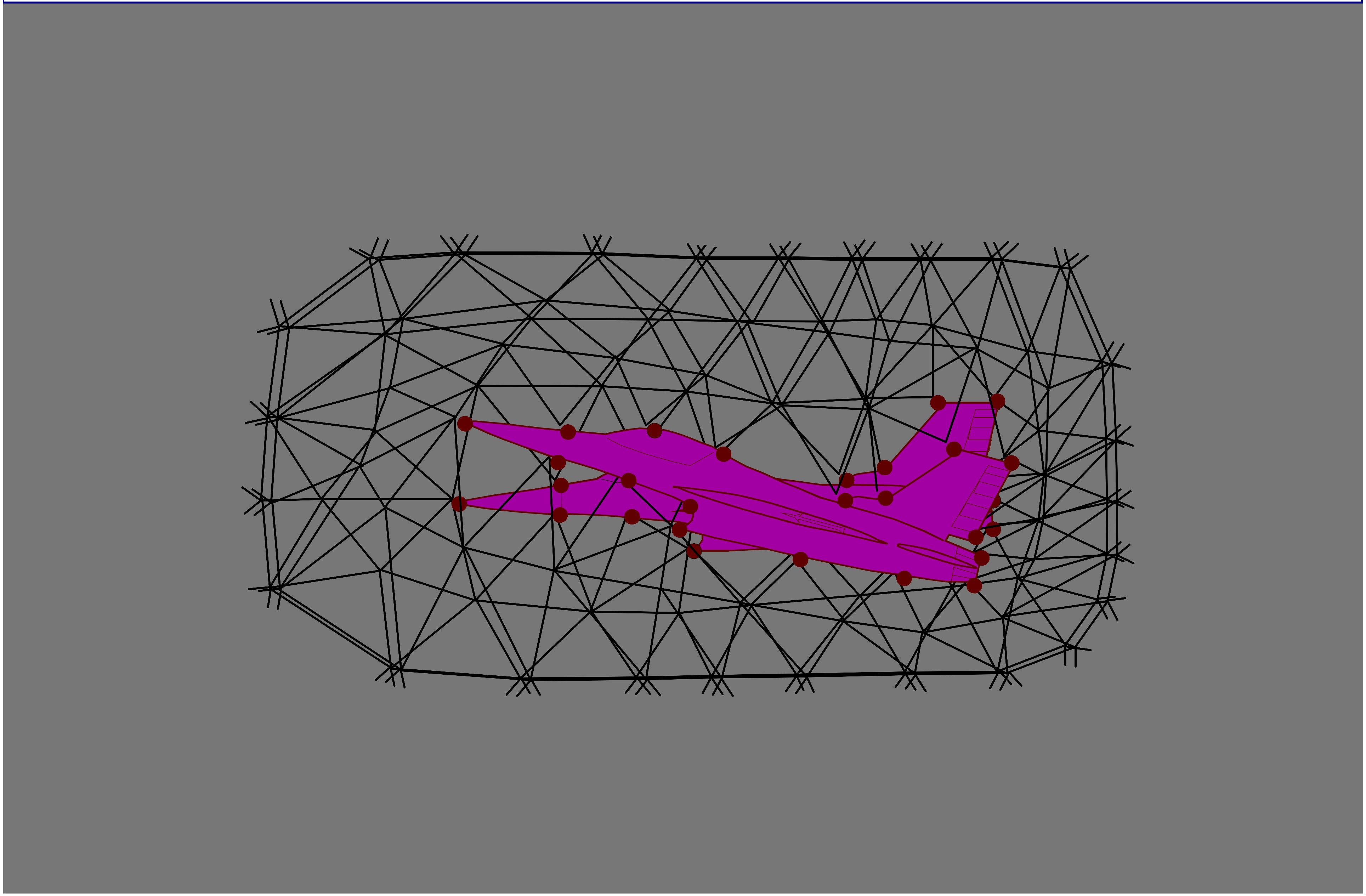
$$\frac{\partial (JW)}{\partial t}\Big|_{\vec{a}} + J\vec{\nabla}_{\vec{x}}.\vec{\mathcal{F}}^{\mathbf{c}}(W) = \frac{1}{Re}J\vec{\nabla}.\vec{\mathcal{R}}(W)$$

$$\vec{\mathcal{F}}^{\mathbf{c}}(W) = \vec{\mathcal{F}}(W) - \frac{dx}{dt}W$$

Dynamic Fluid Mesh

$$\widetilde{\mathbf{M}}\frac{\mathbf{d^2x}}{\mathbf{dt^2}} + \widetilde{\mathbf{D}}\frac{\mathbf{dx}}{\mathbf{dt}} + \widetilde{\mathbf{K}}\,\mathbf{x} = \mathbf{0}$$

FLUID-MESH MOTION



FOUR-FIELD FORMULATION

Structural Dynamics

$$\boxed{\mathbf{M}\frac{\mathbf{d}^2\mathbf{u}}{\mathbf{dt}^2} + \mathbf{f}^{\mathbf{int}}(\mathbf{u}\,,\gamma) - \mathbf{C}\,\boldsymbol{\theta}^{S} = \mathbf{f}^{\mathbf{ext}}\Big(\mathbf{W}(\mathbf{x},\mathbf{t}),\mathbf{x}\,,\gamma\Big)}$$

Heat Transfer

$$\boxed{\mathbf{Q}\,\frac{\mathbf{d}\boldsymbol{\theta}^{\mathbf{S}}}{\mathbf{dt}} + \mathbf{H}\,\boldsymbol{\theta}^{\mathbf{S}} = \mathbf{g}^{\mathbf{ext}}(\mathbf{W})}$$

TRANSMISSION CONDITIONS

STANFORD MECHANICAL ENGINEERING

Fluid / Structure Interface

Momentum

$$\sigma^S \, n = -\,(p - p_{ref})\, n + f$$

Kinematics

$$(v^F - \dot{u})\, n = 0$$

$$\text{or} \quad v^F - \dot{u} = 0$$

Heat transfer

$$\theta^S = \theta^F$$

$$\kappa^S \nabla \theta^S \, n = -\kappa^F \nabla \theta^F \, n$$

Dynamic Fluid Mesh / Structure Interface

Compatibility

$$x = u$$

$$\dot{x} = \dot{u}$$

COMPUTATIONAL RESEARCH

STANFORD MECHANICAL ENGINEERING

- CFD on moving grids
- Exchange of aerodynamic and elastodynamic data
- Coupled solution algorithms

➡ main results

➡ references for further details (see last slide)

CFD ON DYNAMIC MESHES

- ALE Navier-Stokes equations

$$\frac{\partial(JW)}{\partial t}\Big|_{\vec{a}} + J\vec{\nabla}_{\vec{x}}.\vec{\mathcal{F}}^{\mathbf{c}}(W) = \frac{1}{Re} J\vec{\nabla}.\vec{\mathcal{R}}(W)$$

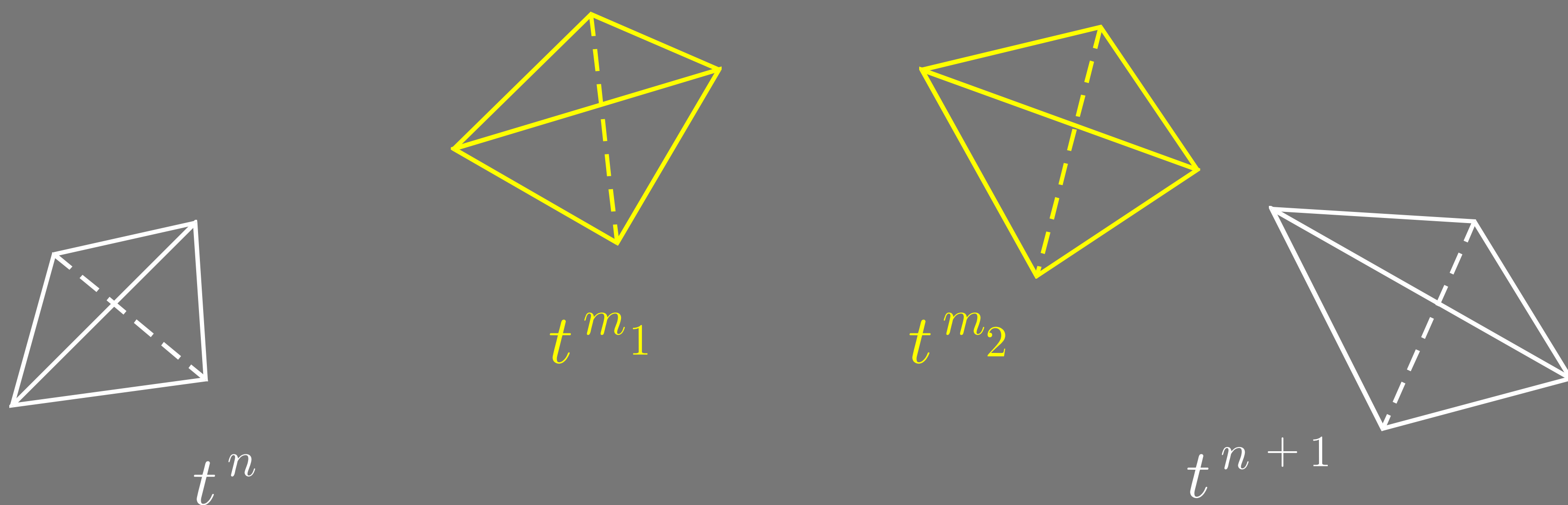
$$\vec{\mathcal{F}}^{\mathbf{c}}(W) = \vec{\mathcal{F}}(W) - \frac{dx}{dt}W$$

- Mesh motion

  - during the first part of this talk, assume that x and therefore $\frac{dx}{dt}$ are given (for example, forced oscillations)

- Advancing the FV or FE flow solution

$$\int_{t^n}^{t^{n+1}} \frac{d}{dt}(\mathbf{V}_i \mathbf{W}_i)\, dt + \int_{t^n}^{t^{n+1}} \mathbf{F}_i^c(\mathbf{W}, \mathbf{x}, \dot{\mathbf{x}})\, dt = \int_{t^n}^{t^{n+1}} \mathbf{R}_i(\mathbf{W}, \mathbf{x})\, dt$$



FLUX TIME-AVERAGING

- p-th order implicit BDF schemes on moving grids

$$\sum_{r=1}^{-p+1} \alpha_{n+r} \mathbf{V}_{\mathbf{i}}(\mathbf{x}^{n+r}) \mathbf{W}_{\mathbf{i}}^{n+r}$$

$$+\Delta t^{n} \sum_{s} \left(w_s^c \, \mathbf{F}_i^c(\mathbf{W}^{n+1}, \mathbf{x}_s^c, \dot{\mathbf{x}}_s^c) - w_s^d \, \mathbf{R}_i(\mathbf{W}^{n+1}, \mathbf{x}_s^d) \right) = 0$$

- $\mathbf{x}_s^c$, $\dot{\mathbf{x}}_s^c$, $\mathbf{x}_s^d$?

- w_s^c, w_s^d ?

FIRST GUIDELINE

Idea: conservation of a uniform flow

$$\frac{\partial(JW)}{\partial t}\Big|_{\vec{a}} + J\vec{\nabla}_{\vec{x}}.\vec{\mathcal{F}}^{\mathbf{c}}(\,W\,) = \frac{1}{Re}J\vec{\nabla}.\vec{\mathcal{R}}(\,W\,)$$

- Discretize in space, set W = W* , then choose a time-discretization scheme that computes exactly the resulting relationship

THE SEMI-DISCRETE GCL

FV: $$\mathbf{V}_i(\mathbf{x}^{n+1}) - \mathbf{V}_i(\mathbf{x}^{n}) = \int_{t^n}^{t^{n+1}} \int_{\partial C_i(\mathbf{x})} \dot{\mathbf{x}} \; \mathbf{n} \, ds dt$$

FE: $$\int_{\Omega(t^{n+1})} \mathbf{V^h} \, d\Omega - \int_{\Omega(t^{n})} \mathbf{V^h} \, d\Omega = \int_{t^n}^{t^{n+1}} \int_{\Omega(t)} \mathbf{V^h}_{,\mathbf{i}} \, \dot{\mathbf{x}}_{\mathbf{i}} \, d\Omega \, dt$$

- involve only geometric quantities
- universal (for a given semi-discretization method) geometric conservation laws (GCLs)
- *are independent of any time-integration scheme*

THE p-th ORDER DISCRETE GCL

- Flux time-averaging

$$\sum_{r=1}^{-p+1} \alpha_{n+r} \mathbf{V}_{\mathbf{i}}(\mathbf{x}^{n+r}) \mathbf{W}_{\mathbf{i}}^{n+r}$$

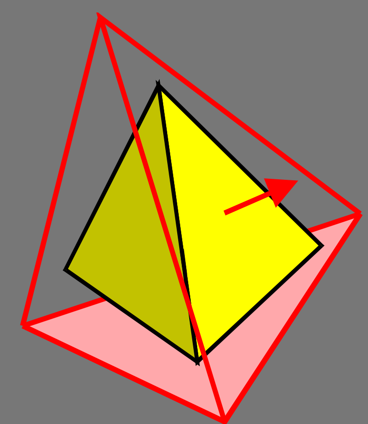
$$+\Delta t^{n} \sum_{s} \Big(w_s^{c}\, \mathbf{F}_i^{c}(\mathbf{W}^{n+1}, \mathbf{x}_s^{c}), \dot{\mathbf{x}}_s^{c} \Big) - w_s^{d}\, \mathbf{R}_i\big(\mathbf{W}^{n+1}, \mathbf{x}_s^{d}\big)\Big) = 0$$

- Set W = W* = constant

THE p-th ORDER DGCL

$$\sum_{r=1}^{-p+1} \alpha_{n+r} \mathbf{V}_{\mathbf{i}}(\mathbf{x}^{n+r}) = \Delta t^n \sum_s w_s^c \, \mathbf{G}_i(\mathbf{x}_s^c, \dot{\mathbf{x}}_s^c)$$

p-th order *Discrete*
Geometric Conservation Law
(DGCL)



characterizes the time-integrator of interest
(there is NO universal DGCL)

APPLICATION TO SCHEME DESIGN

- p-th order DGCL

$$\sum_{r=1}^{-p+1} \alpha_{n+r} \mathbf{V}_{\mathbf{i}}(\mathbf{x}^{n+r}) = \Delta t^{n} \sum_{s} w_s^c \ \mathbf{G}_i(\mathbf{x}_s^c, \dot{\mathbf{x}}_s^c)$$

- can be used to determine $\mathbf{x}_s^c$, $\dot{\mathbf{x}}_s^c$, and w_s^c
- does not determine $\mathbf{x}_s^d$ and w_s^d
- *unknown order of time-accuracy of resulting scheme*

FV EXAMPLE

- 2^{nd}-order implicit BDF satisfying its 2^{nd}-DGCL

$$\begin{cases} \mathbf{x}^c_{1(3)} = \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n+1\ (n)} + \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n\ \ (n\text{-}1)} \\ \mathbf{x}^c_{2(4)} = \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n+1\ (n)} + \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n\ \ (n\text{-}1)} \\ \dot{\mathbf{x}}^c_1 = \dot{\mathbf{x}}^c_2 = \frac{\mathbf{x}^{n+1} - \mathbf{x}^n}{\boldsymbol{\Delta}\mathbf{t}} \qquad \dot{\mathbf{x}}^c_3 = \dot{\mathbf{x}}^c_4 = \frac{\mathbf{x}^n - \mathbf{x}^{n\text{-}1}}{\boldsymbol{\Delta}\mathbf{t}} \\ w^c_1 = w^c_2 = \frac{3}{2} \qquad w^c_3 = w^c_4 = -\frac{1}{2} \end{cases}$$

APPLICATION TO SCHEME DESIGN

STANFORD MECHANICAL ENGINEERING

- p-th order DGCL

$$\sum_{r=1}^{-p+1} \alpha_{n+r} \mathbf{V}_{\mathbf{i}}(\mathbf{x}^{n+r}) = \Delta t^{n} \sum_{s} w_s^c \; \mathbf{G}_i(\mathbf{x}_s^c, \dot{\mathbf{x}}_s^c)$$

- can be used to determine $\mathbf{x}_s^c$, $\dot{\mathbf{x}}_s^c$, and w_s^c
- does not determine $\mathbf{x}_s^d$ and w_s^d
- *unknown order of time-accuracy of resulting scheme*

SECOND GUIDELINE

STANFORD MECHANICAL ENGINEERING

Idea: preserving the order of time-accuracy on fixed grids of the original scheme

- Application to the 2nd-order implicit BDF

$$\mathbf{x}^{c} = \zeta^{n+1}\,\mathbf{x}^{n+1} + \zeta^{n}\,\mathbf{x}^{n} + (\mathbf{1} - \zeta^{n} - \zeta^{n+1})\,\mathbf{x}^{n-1}$$

$$\dot{\mathbf{x}}^{c} = \theta^{n+1}\,\mathbf{x}^{n+1} + \theta^{n}\,\mathbf{x}^{n} + (\mathbf{1} - \theta^{n} - \theta^{n+1})\,\mathbf{x}^{n-1}$$

→ Taylor expansion to evaluate truncation error

A SIMPLE 2nd-ORDER SCHEME

- The one-point rule

$$\left\{\begin{aligned} \mathbf{x}^c &= \mathbf{x}^{n+1} \\ \dot{\mathbf{x}}^c &= \frac{3}{2\Delta t}\mathbf{x}^{n+1} - \frac{2}{\Delta t}\mathbf{x}^n + \frac{1}{2\Delta t}\mathbf{x}^{n-1} \\ w_1^c &= 1 \qquad w_2^c = w_3^c = w_4^c = 0 \end{aligned}\right.$$

$$\left\{\begin{aligned} \mathbf{x}^d &= \mathbf{x}^{n+1} \\ w_1^d &= 1 \qquad w_2^d = w_3^d = w_4^d = 0 \end{aligned}\right.$$

→ two flux evaluations per time-step

ANOTHER 2nd-ORDER SCHEME

STANFORD MECHANICAL ENGINEERING

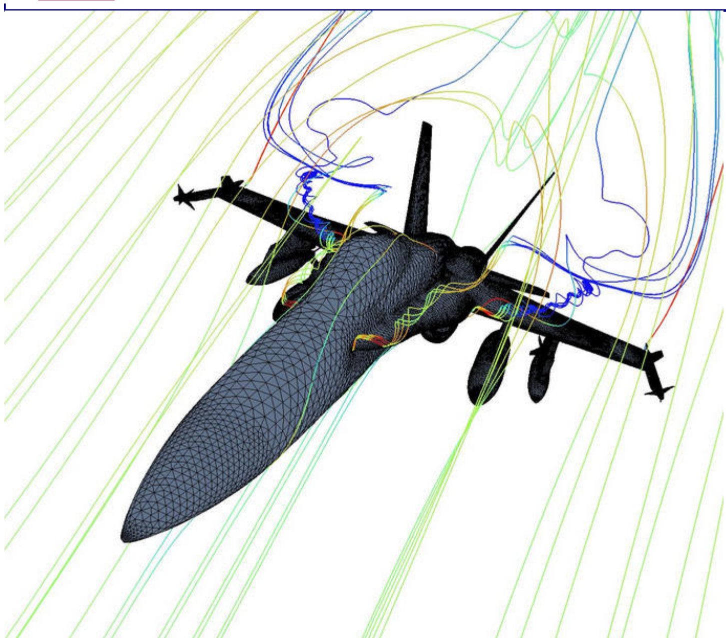
- The four/one-point rule (satisfies 2nd-DGCL!)

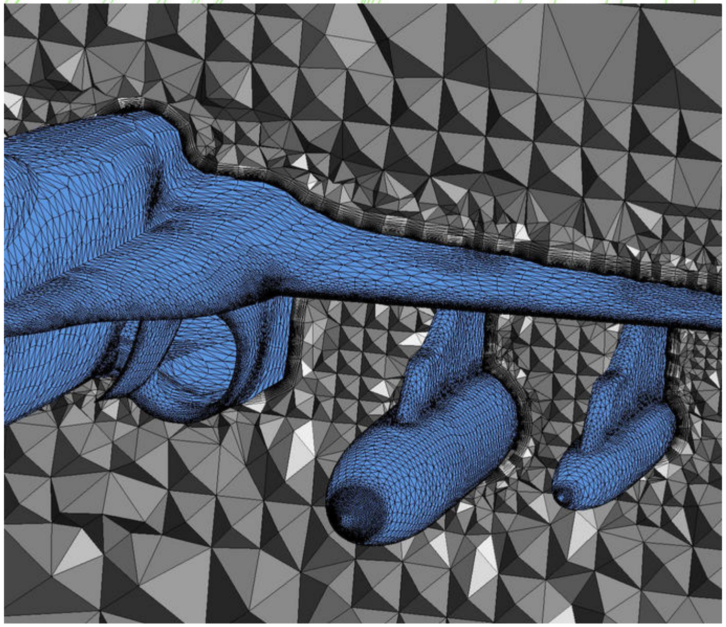
$$\left\{\begin{aligned}
\mathbf{x}^c_{1(3)} &= \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n+1\ (\mathbf{n})} + \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n\ \ (n\text{-}1)} \\
\mathbf{x}^c_{2(4)} &= \frac{1}{2}\left(1+\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n+1\ (\mathbf{n})} + \frac{1}{2}\left(1-\frac{1}{\sqrt{3}}\right)\mathbf{x}^{n\ \ (n\text{-}1)} \\
\dot{\mathbf{x}}^c_1 &= \dot{\mathbf{x}}^c_2 = \frac{\mathbf{x}^{n+1} - \mathbf{x}^n}{\Delta \mathbf{t}} \qquad \dot{\mathbf{x}}^c_3 = \dot{\mathbf{x}}^c_4 = \frac{\mathbf{x}^{n} - \mathbf{x}^{n-1}}{\Delta \mathbf{t}} \\
w^c_1 &= w^c_2 = \frac{3}{2} \qquad w^c_3 = w^c_4 = -\frac{1}{2}
\end{aligned}\right.$$

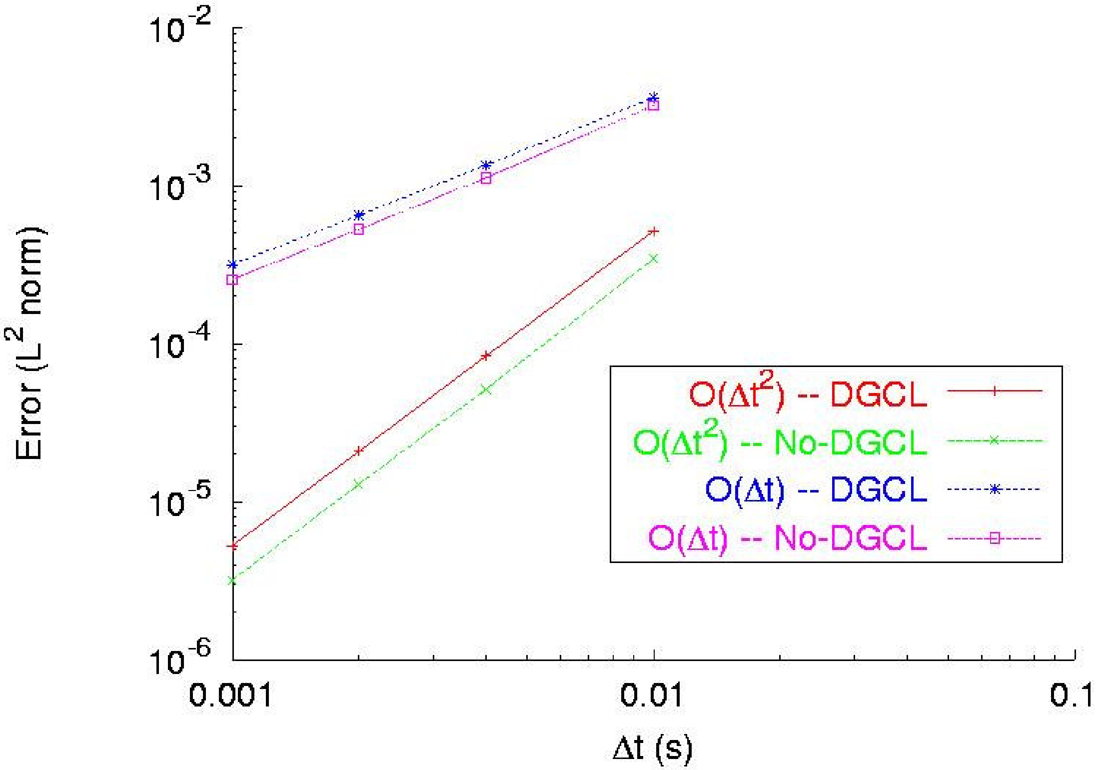
$$\left\{\begin{aligned}
\mathbf{x}^d &= \mathbf{x}^{n+1} \\
w^d_1 &= 1 \qquad w^d_2 = w^d_3 = w^d_4 = 0
\end{aligned}\right.$$

→ five flux evaluations per time-step

REALIZATION







TO DGCL OR NOT TO DGCL

- The one-point rule (FV)
  - two flux computations per time-step
  - 2nd-order time-accurate
  - violates its corresponding (2nd-order) DGCL
- The four-point rule (FV)
  - five flux computations per time-step
  - 2nd-order time-accurate
  - satisfies its corresponding (2nd-order) DGCL
- Which is better? The more economical one?

CONFIGURATION AVERAGING

STANFORD MECHANICAL ENGINEERING

- FV method, 2nd-order implicit BDF

$$\sum_{r=1}^{-1} \alpha_{n+r} \mathbf{V_i}(\mathbf{x}^{n+r}) \mathbf{W}_{\mathbf{i}}^{n+r}$$

$$+\Delta t^{n} \left(\mathbf{F}_i^c \left(\mathbf{W}^{n+1}, \sum_{s=1}^{4} w_s^c \vec{\boldsymbol{\nu}}_{ij}^s , \sum_{s=1}^{4} w_s^c \boldsymbol{\kappa}_{ij}^s \right) - \mathbf{R}_i \left(\mathbf{W}^{n+1}, \sum_{s=1}^{4} w_s^c x^s \right) \right) = 0$$

- two flux evaluations per time-step
- 2nd-order time-accurate
- satisfies its DGCL

A BRIEF HISTORY

- The terminology ``geometric conservation law" was coined in 1979 by Thomas and Lombard
  (finite differencing, mass conservation)

- The computational method proposed in 1959 by Godunov incorporated a similar requirement

UNTIL RECENTLY

- Recurrent conflicting assertions in the literature about the practical usefulness of the "[D]GCL"
- Theoretical status of this "requirement" is unclear (after all, why should one pay special attention to a uniform flow field?)
- Why the constant solution of the Navier-Stokes equations must be computed exactly by a given numerical scheme, while the other solutions are only approximated by that scheme?

UNFORTUNATE CONFUSIONS

- Frequent confusions

  - the continuous GCL

$$\mathbf{V}_i(x^{n+1}) - \mathbf{V}_i(x^n) = \int_{t^n}^{t^{n+1}} \int_{\partial C_i(\mathbf{x})} \dot{x} \;\; \mathbf{n}\, ds dt$$

  - the first-order DGCL

$$\mathbf{V}_i(\mathbf{x}^{n+1}) - \mathbf{V}_i(\mathbf{x}^n) = \Delta t^n \sum_s w_s^c \;\; \mathbf{G}_i(\mathbf{x}_s^c, \dot{\mathbf{x}}_s^c)$$

  - the p-th order DGCL

$$\sum_{r=1}^{-p+1} \alpha_{n+r} \mathbf{V}_{\mathbf{i}}(\mathbf{x}^{n+r}) = \Delta t^n \sum_s w_s^c \;\; \mathbf{G}_i(\mathbf{x}_s^c, \dot{\mathbf{x}}_s^c)$$

A SUFFICIENT CONDITION

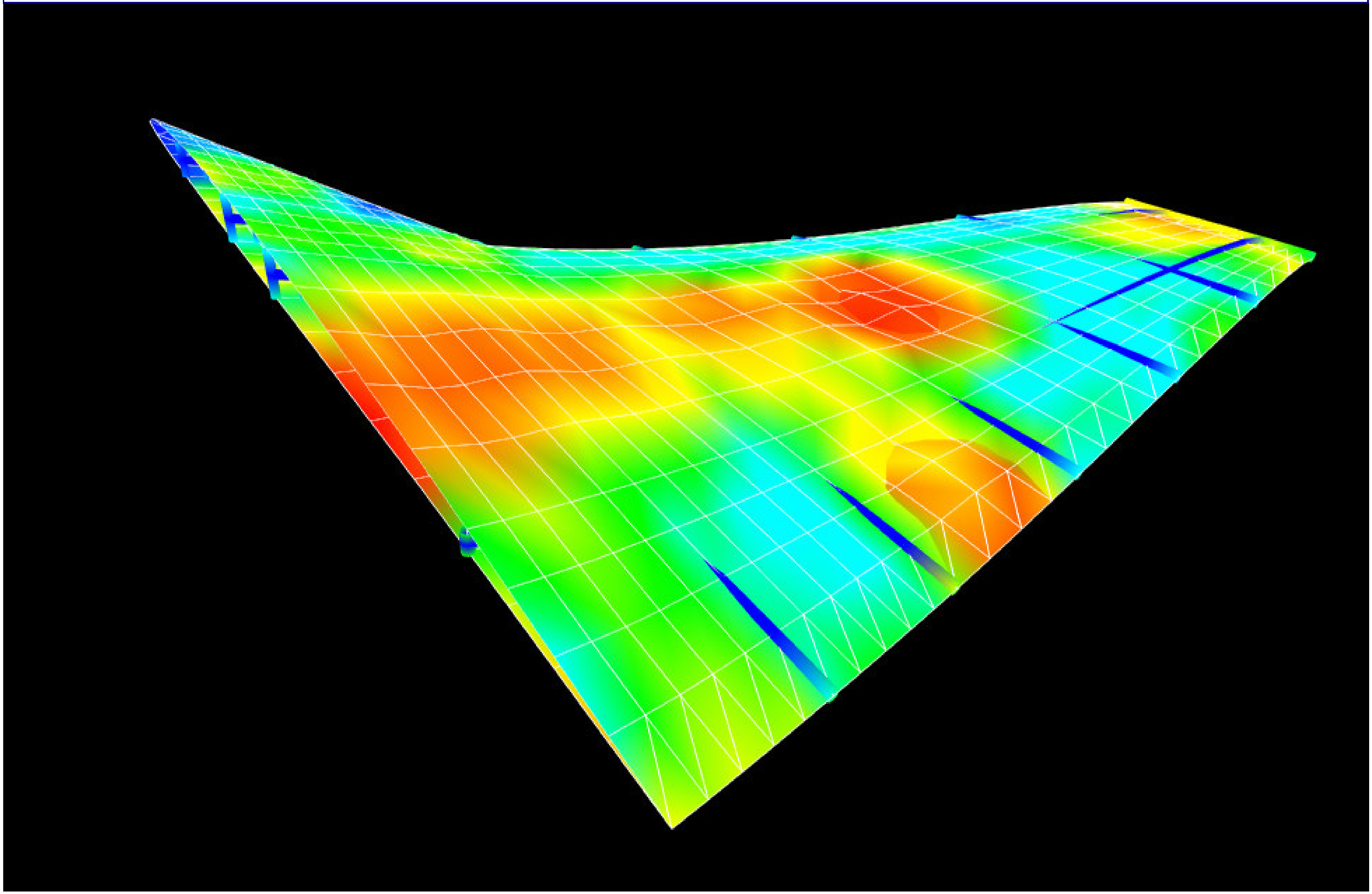
- Proposition 1 (Farhat & Guillard, 1998)

  For a given scheme that is p-th order time-accurate on a fixed mesh, satisfying the corresponding p-th order DGCL is a sufficient condition for this scheme to be at least 1st-order time-accurate on a moving mesh

OTHER SURPRISING STATEMENTS

- The "[D]GCL" is neither a sufficient nor a necessary condition for numerical stability …

RELATIONSHIP TO STABILITY



NECESSARY & SUFFICIENT CONDITION

- Proposition 2 (Farhat & Grandmont, 1999)

  Given a numerical scheme with established nonlinear stability properties (i.e. unconditionally stable) on a fixed mesh, satisfying the corresponding pth-order DGCL is a necessary and sufficient condition for preserving these numerical *nonlinear stability* properties (discrete maximum principle) on a moving mesh

(Nonlinear scalar conservation law and the θ family of schemes)

STABILITY ESTIMATES

- Proposition 3 (Farhat & Grandmont, 2001)

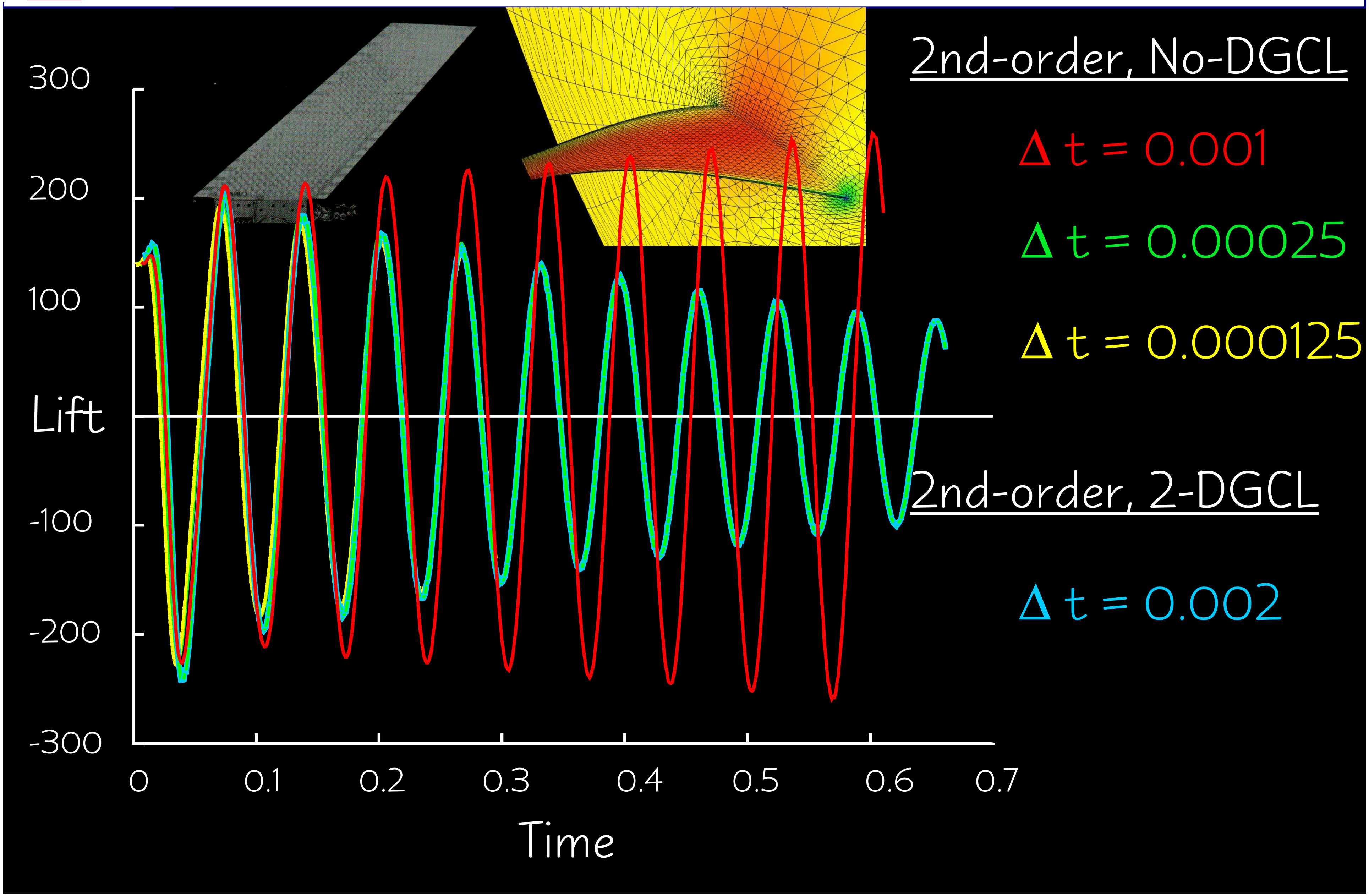
Consider an extension to moving grids of the classical θ-scheme. If this extension violates its DGCL, then the following stability estimates hold:

Explicit case $$\|\mathbf{W}^n\|_\infty \leq \|\mathbf{W}^0\|_\infty \; e^{C\Delta t\, T}$$

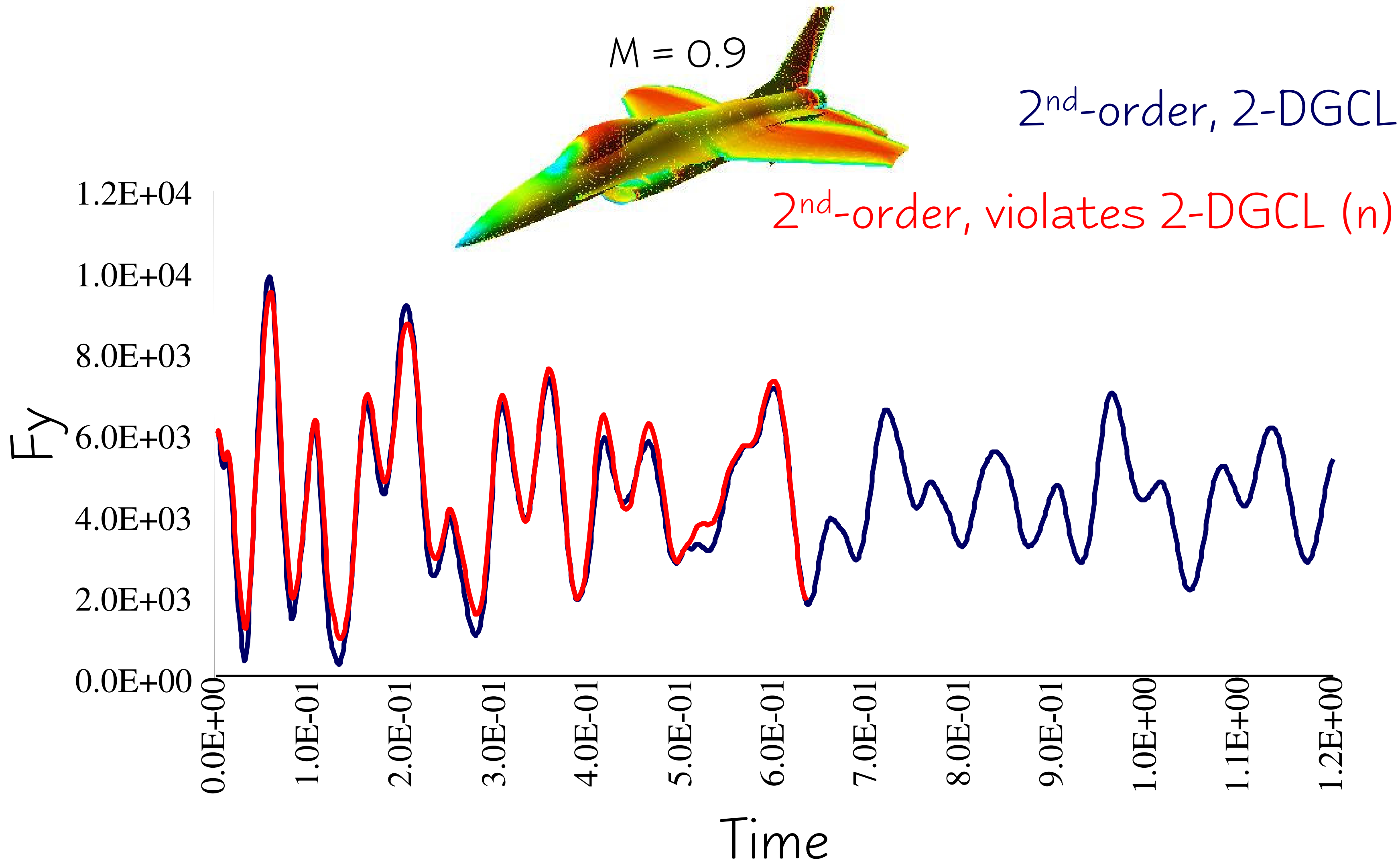
Implicit case $$\|\mathbf{W}^n\|_\infty \leq \|\mathbf{W}^0\|_\infty \; e^{\frac{C\Delta t\, T}{1-C\Delta t^2}}$$

where the constant C depends on the velocity of the grid

SIGNIFICANCE



SIGNIFICANCE



COUPLED SOLUTION ALGORITHMS

- Monolithic schemes

$$\begin{pmatrix} \text{Fluid subsystem} \\ \text{Structure subsystem} \\ \text{Dynamic fluid mesh subsystem} \end{pmatrix} \begin{pmatrix} W \\ u \\ x \end{pmatrix}$$

- Context

  - transient (time-dependent, unsteady, ...) and NOT algebraic (steady) problems

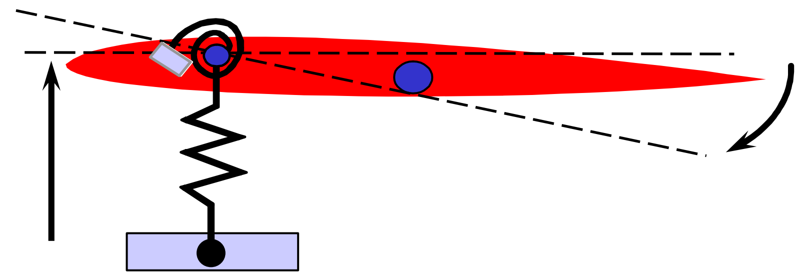
COUPLED SOLUTION ALGORITHMS

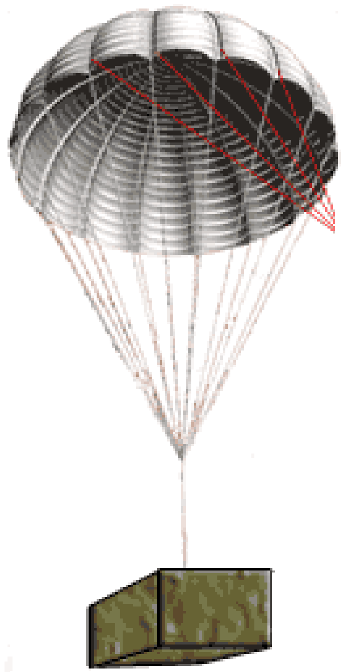
- Monolithic schemes

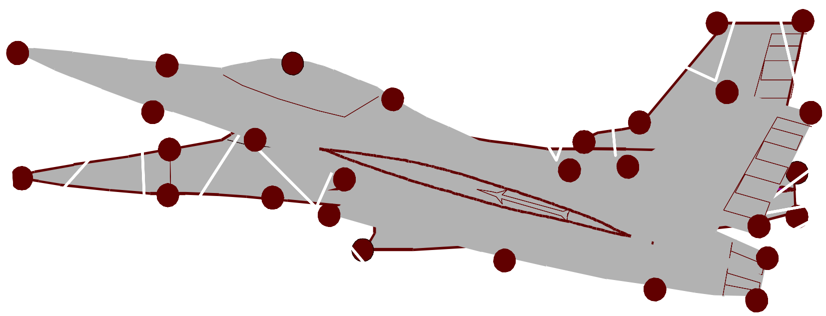
$$\begin{pmatrix} \text{Fluid subsystem} \\ \text{Structure subsystem} \\ \text{Dynamic fluid mesh subsystem} \end{pmatrix} \begin{pmatrix} W \\ u \\ x \end{pmatrix}$$

- re-formulation of structure problem as a 1st-order ODE
- conversion of a system of ODEs into a macro DAE
- 2/3rd-monolithic formulation
- limited opportunities for code re-use
- algebra-type parallelism

MONOLITHIC SCHEMES







PARTITIONED SCHEMES

- Partitioned (staggered) schemes
  - off-the-shelf schemes
  - different numerics for different physics
  - subcycling
  - inter-parallelism
  - software modularity
  - …

  * Loosely-coupled scheme

    (+ inner-iterations

    = strongly-coupled scheme)

SOME MISCONCEPTIONS (for Class III)

- Loosely-coupled schemes are
  - inaccurate
  - unstable for any realistic time-step
  - unconditionally unstable when the mass of the fluid subsystem is much greater than the mass of the structure subsystem
  - useless (not to say stupid)

- Inner-iterations improve accuracy

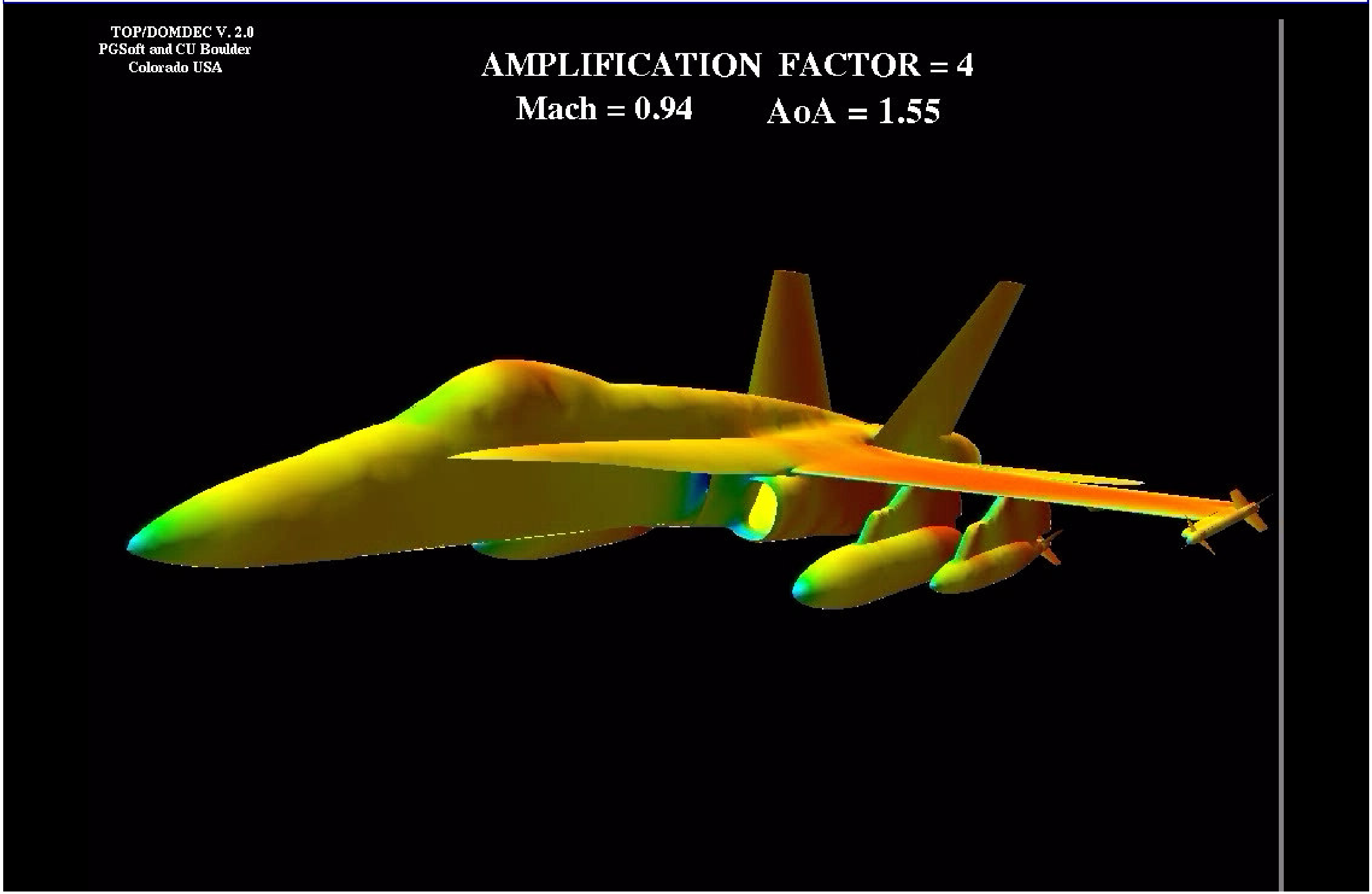
BACKGROUND

- Peacemann and Rachford (1955)
  - ADI, LOD, AFM
  - implicit one dimension at a time
  - desired accuracy and stability can be maintained for many problems of interest

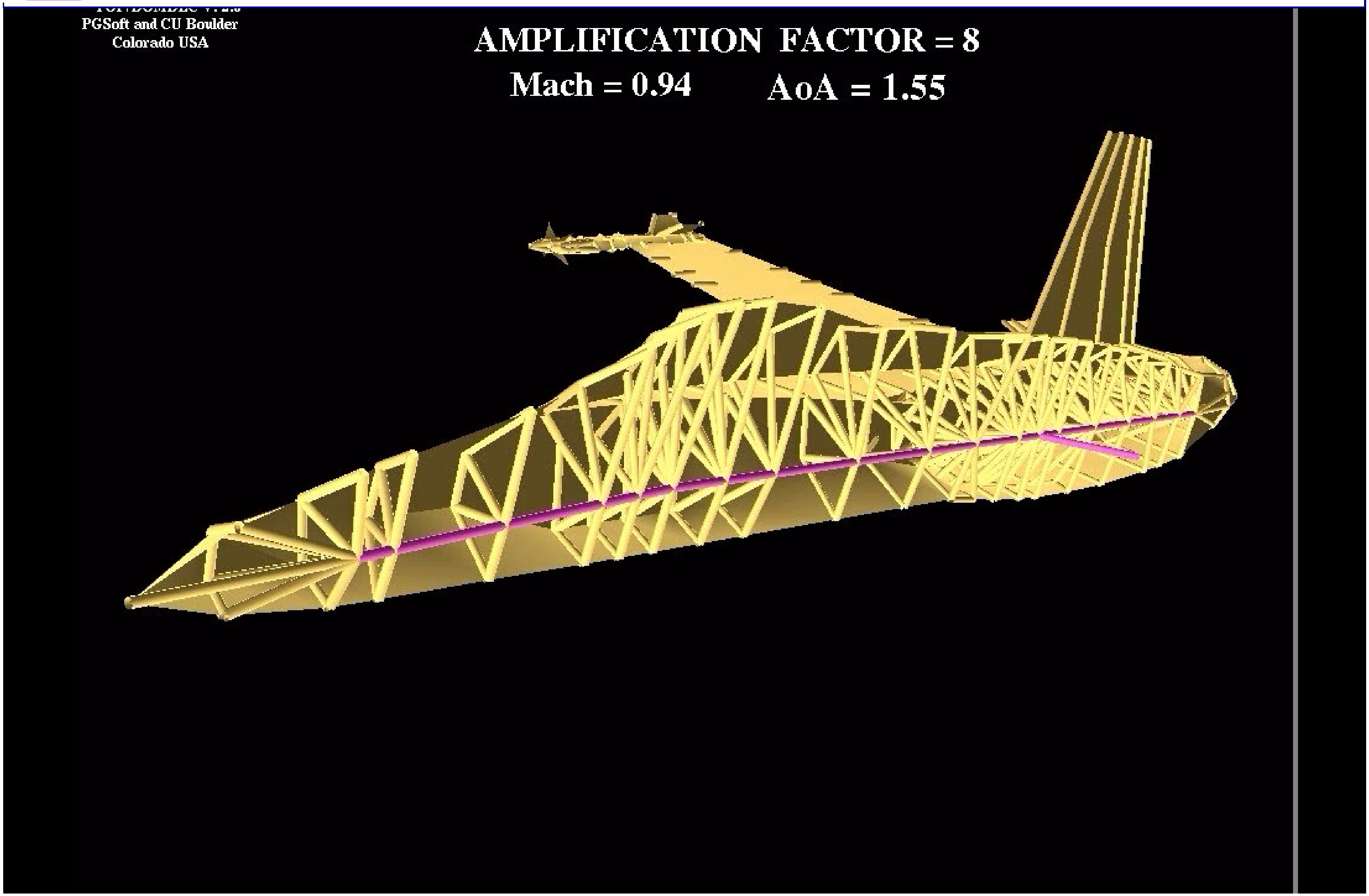
- Park, Felippa & DeRuntz (1977-1983)
  - acoustic pressure
  - desired accuracy and stability can be maintained by introducing prediction, augmentation, …

→ ART

WHAT'S THAT?

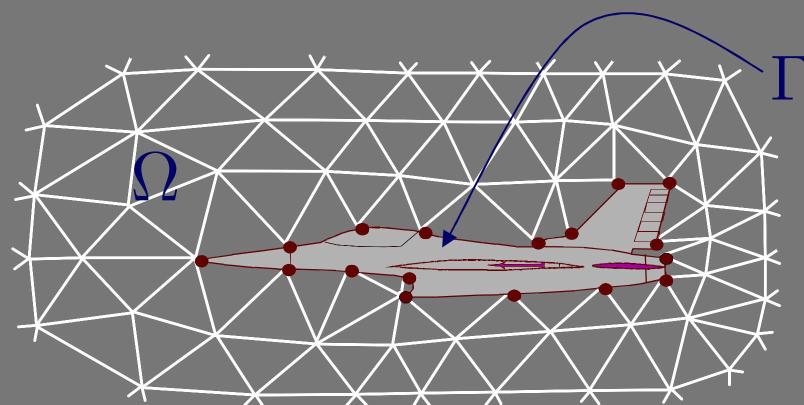

TOP/DOMDEC V. 2.0
PGSoft and CU Boulder
Colorado USA
AMPLIFICATION FACTOR = 4
Mach = 0.94
AoA = 1.55

CAUSE OR CONSEQUENCE?



FLUID MESH MOTION

$$K^*_{\Omega\Omega}(t)\,\dot{x}_{\Omega}(t) = -K^*_{\Omega\Gamma}(t)\,\dot{u}_{\Gamma}(t)$$



$$x_\Gamma - x_\Gamma(0) = u_\Gamma$$

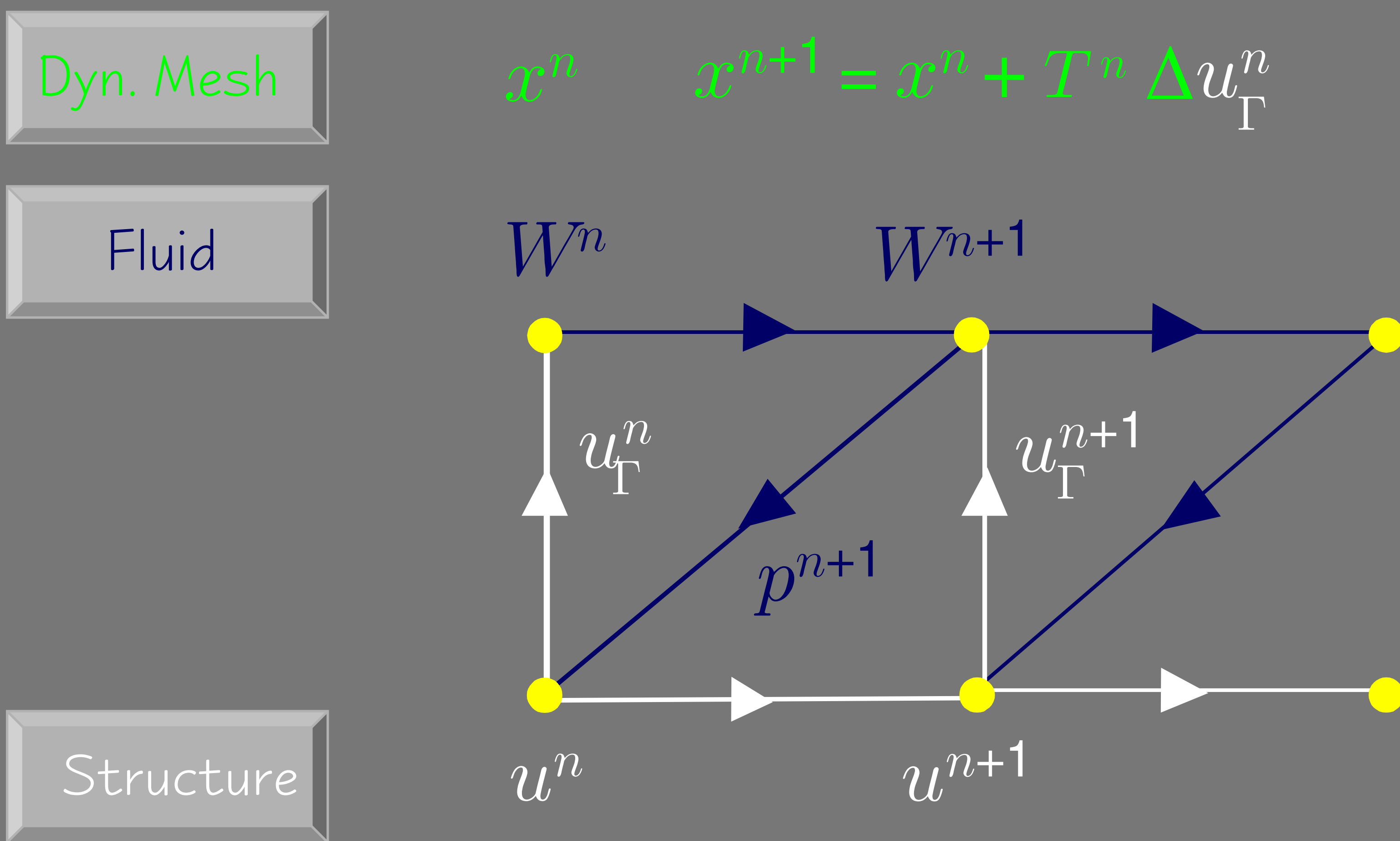
$$\dot{x}_\Gamma = \dot{u}_\Gamma$$

$$x(t) = x(\tau) + \int_\tau^t T(\eta)\,\dot{u}_\Gamma(\eta)\,d\eta$$

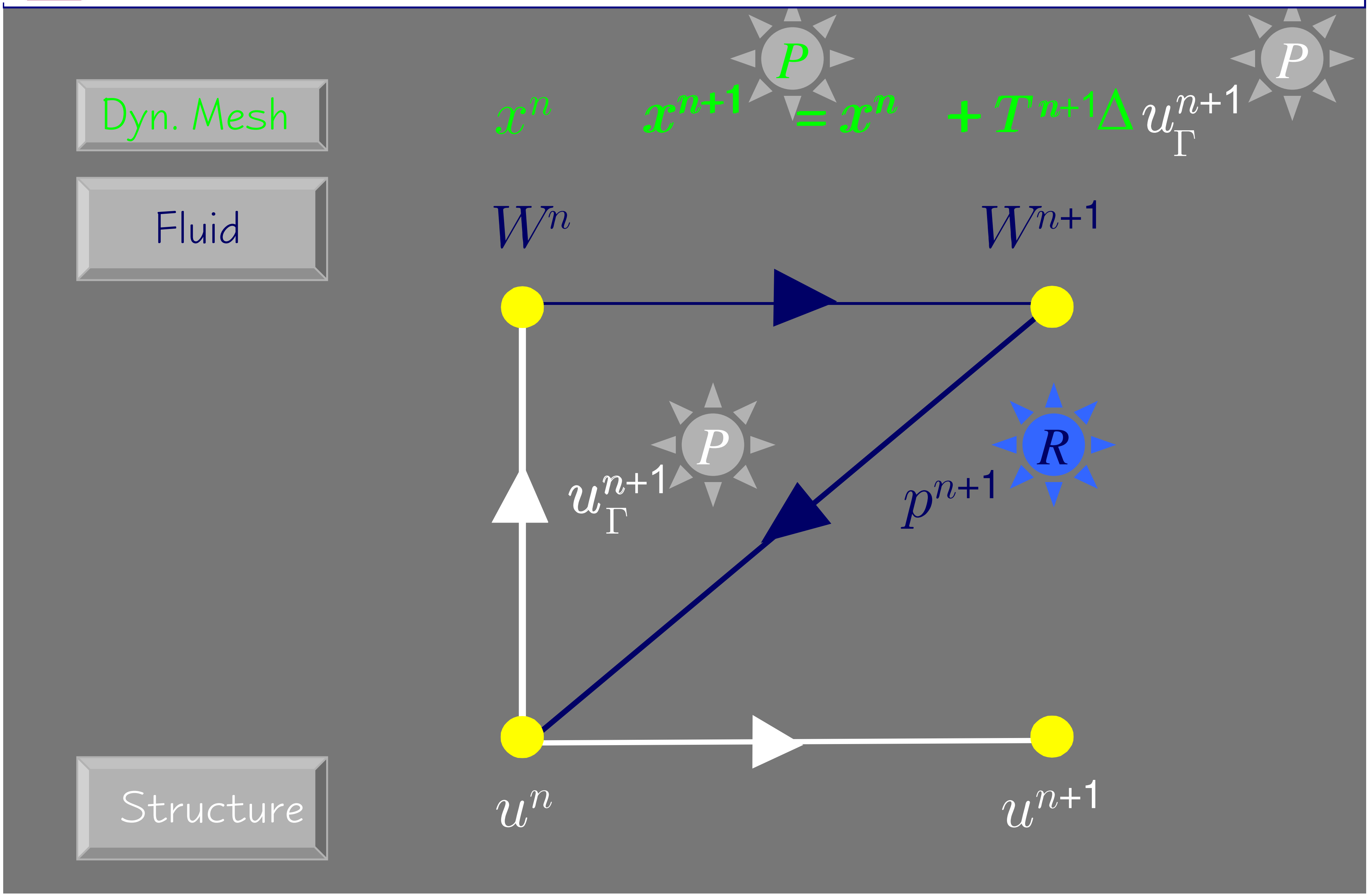
$$T(t) = \begin{vmatrix} -K^{*-1}_{\Omega\Omega}(t)\,K^*_{\Omega\Gamma}(t) \\ I \end{vmatrix} T_\Gamma$$

PARTITIONED SCHEMES

- The "off-the-shelf" loosely-coupled approach



STATE-OF-THE-ART LOOSELY COUPLED



PREDICTORS & RECONSTRUCTORS

- Predictors

$$u^{n+1\,P} = u^n + \alpha_0 \Delta t\, \dot{u}^n + \alpha_1 \Delta t\, (\dot{u}^n - \dot{u}^{n-1})$$

- Reconstructors

$${p^{n+1}}^R = p^n \qquad \text{(Asynchronous)}$$

$${p^{n+1}}^R = p^{n+1} \qquad \text{(Gauss-Seidel)}$$

$${p^{n+1}}^R = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} p(t)\, dt \qquad \text{(Average)}$$

$${p^{n+1}}^R = 2p^{n+1} - p^n \qquad \text{(Conservation of Momentum)}$$

CONTROL PARAMETERS

STANFORD MECHANICAL ENGINEERING

- Predictor: $\alpha_0 , \ \alpha_1$
- Mesh integrator: T^{n+1}
- Reconstructor: ${p^{n+1}}^R$

⇒ provable control of accuracy and numerical stability

EFFECT OF FLUID MESH MOTION

STANFORD MECHANICAL ENGINEERING

Lemma 1

The local truncation error of the time-averaged ALE version of the 3-point BDF scheme implemented in the generalized loosely coupled staggered procedure satisfies

$$\psi_w(t^{n+1}) = \Delta t \sum_{-1}^{1} O(\|x^P(t^{n+k}) - x(t^{n+k})\|) + O(\Delta t^3)$$

Forced fluid-mesh motion and structure-induced fluid-mesh motion do not have the same effect on the accuracy of the ALE flow solver

Lemma 2

The local truncation error of the midpoint rule applied to the structure subproblem satisfies

$$\psi_v(t^{n+1}) = \Delta t \sum_0^1 O\Big(\| f_S{}^{ae}\big(x_\Gamma(t^{n+k})\big) - f_S{}^{ae}\big(x_\Gamma^P(t^{n+k})\big)\|\Big)$$

$$+ O(\Delta t^3)$$

A 2nd-ORDER SoA-LC SCHEME

- Proposition 4 (Farhat, 2004)

If the generalized loosely coupled scheme is equipped with a 2nd-order structure predictor ($\alpha_0 = 1$, $\alpha_1 = ½$), and the matrix T characterizing the fluid-mesh motion algorithm is evaluated as follows

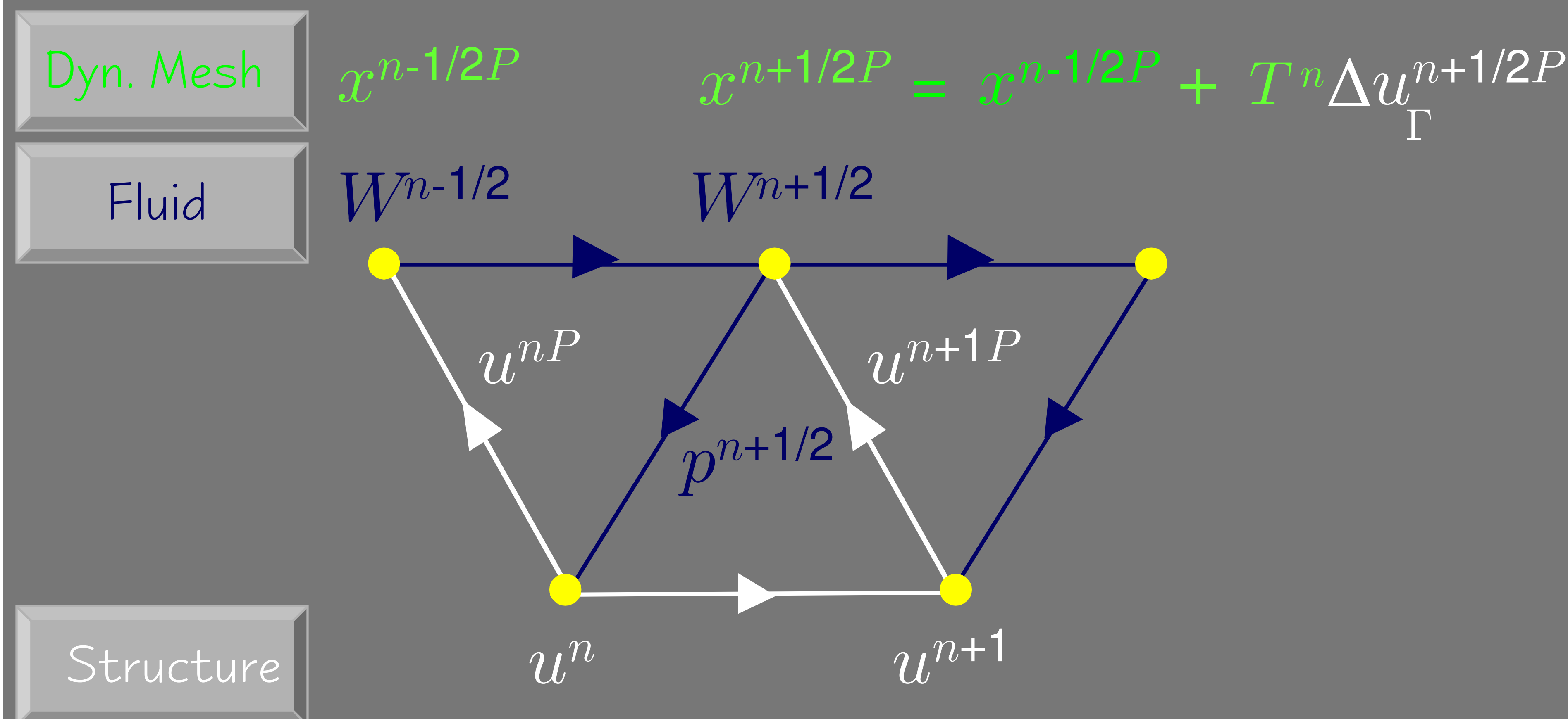
$$T = (T^{n\text{-}1} + T^{n})/2$$

then this scheme is formally 2nd-order time-accurate

$$x^{n+1P} = x^{nP} + \left((T^{n\text{-}1} + T^{n})/2\right)\Delta u_{\Gamma}^{n+1P}$$

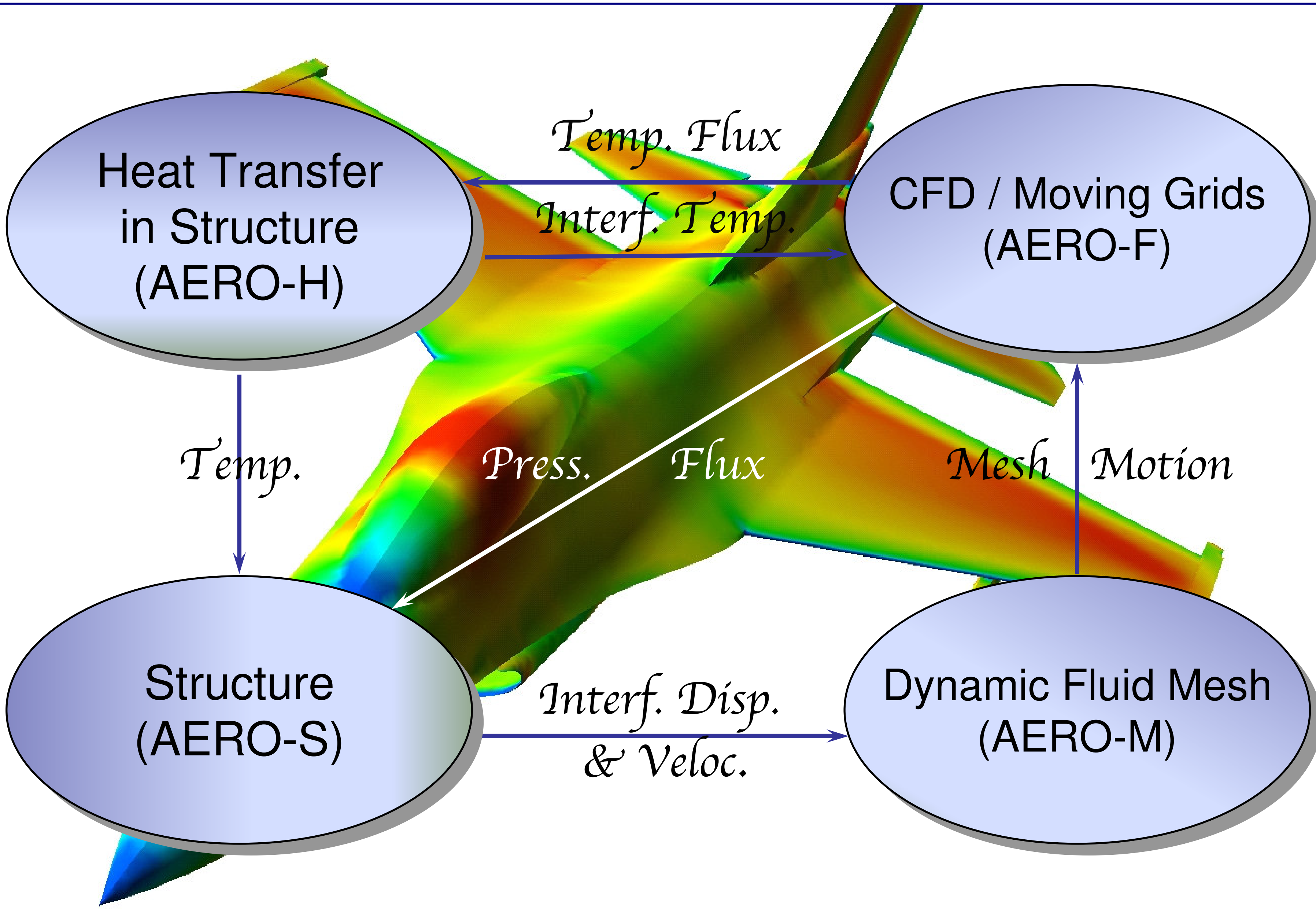
ANOTHER 2nd-ORDER SoA-LC SCHEME

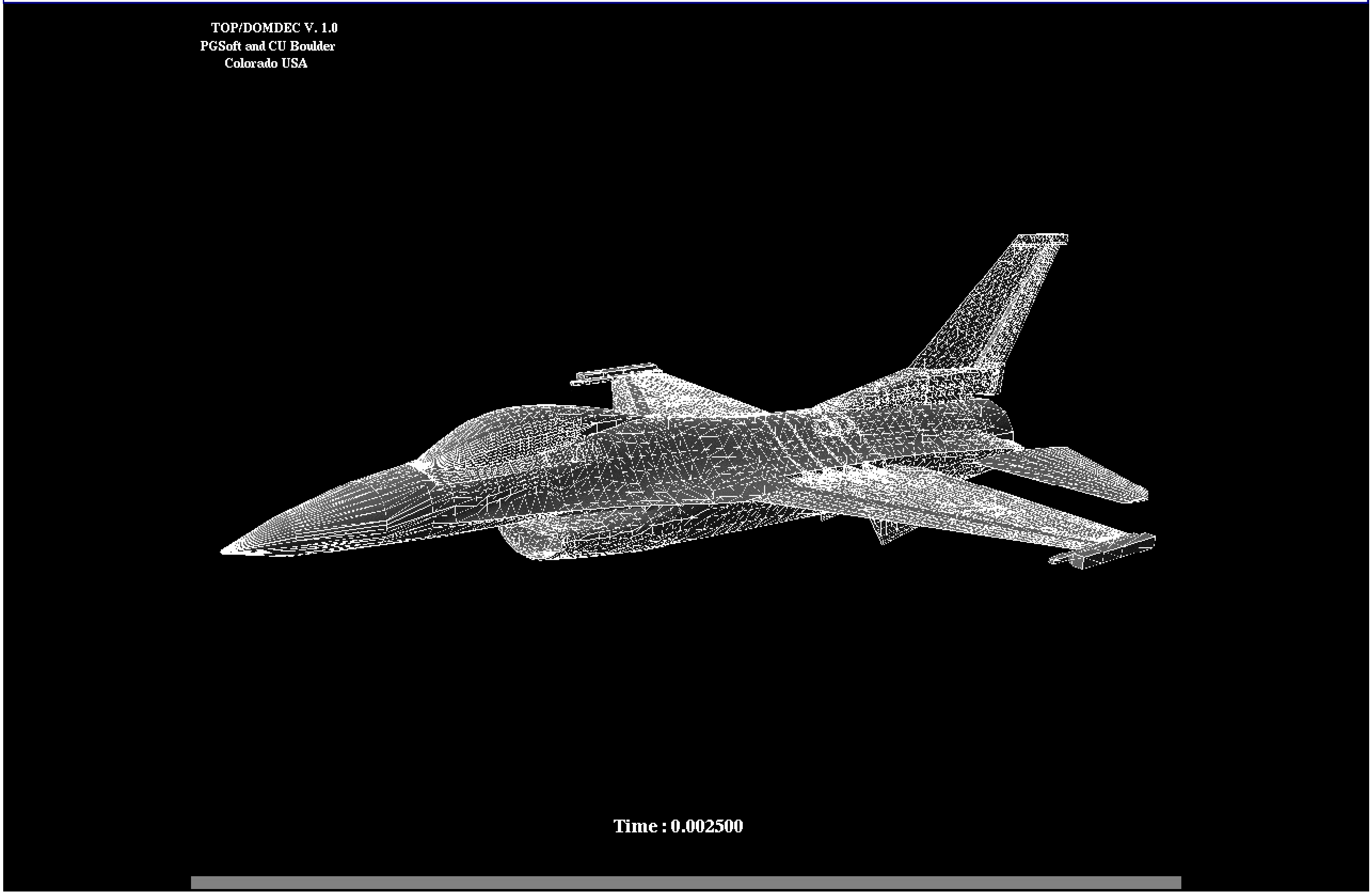
- Proposition 5 (Farhat, 2004)



$$u^{n+1/2P} = u^n + \frac{\Delta t}{2}\dot{u}^n + \frac{\Delta t}{8}(\dot{u}^n - \dot{u}^{n-1})$$

THE AERO PLATFORM

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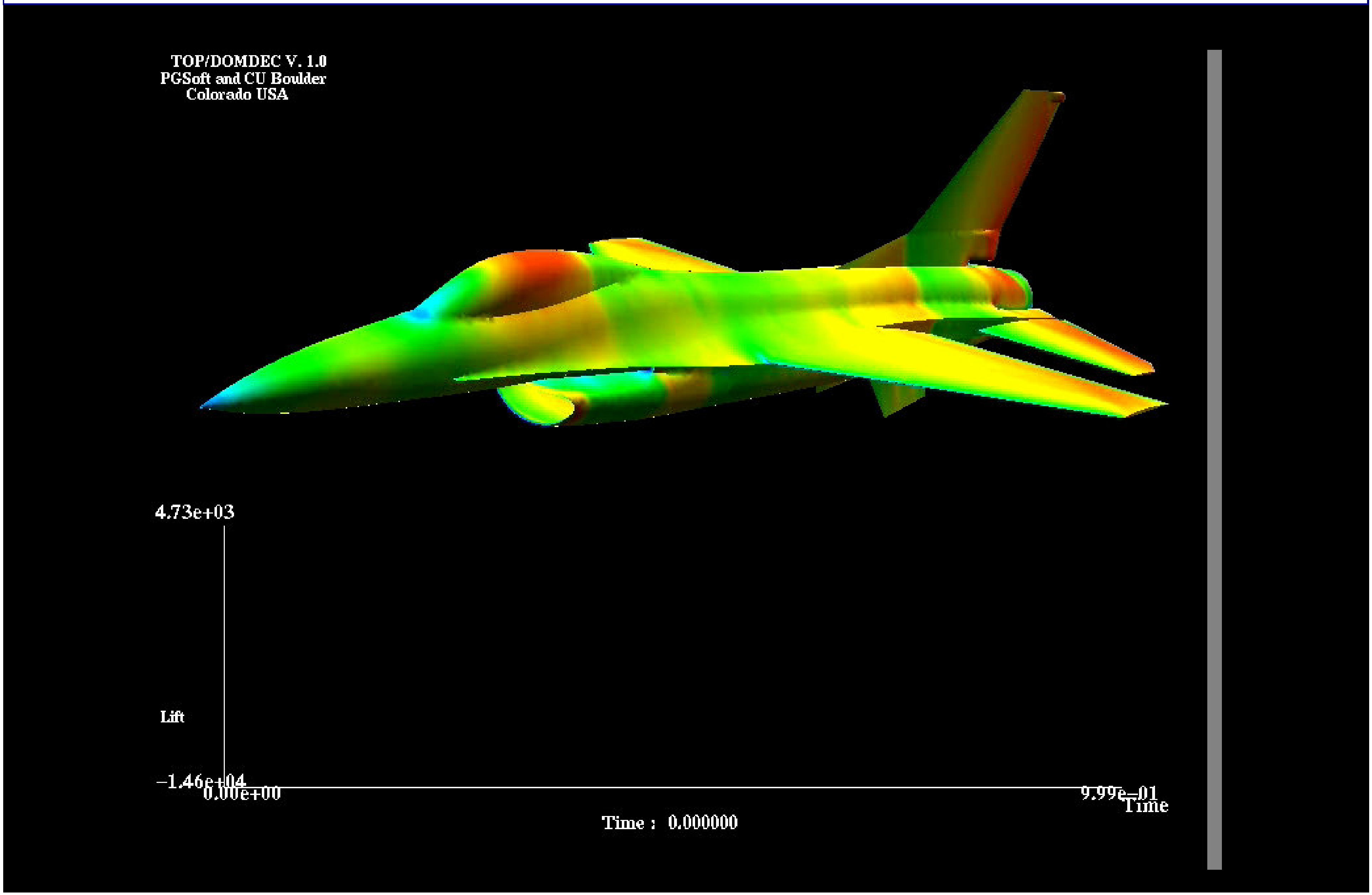
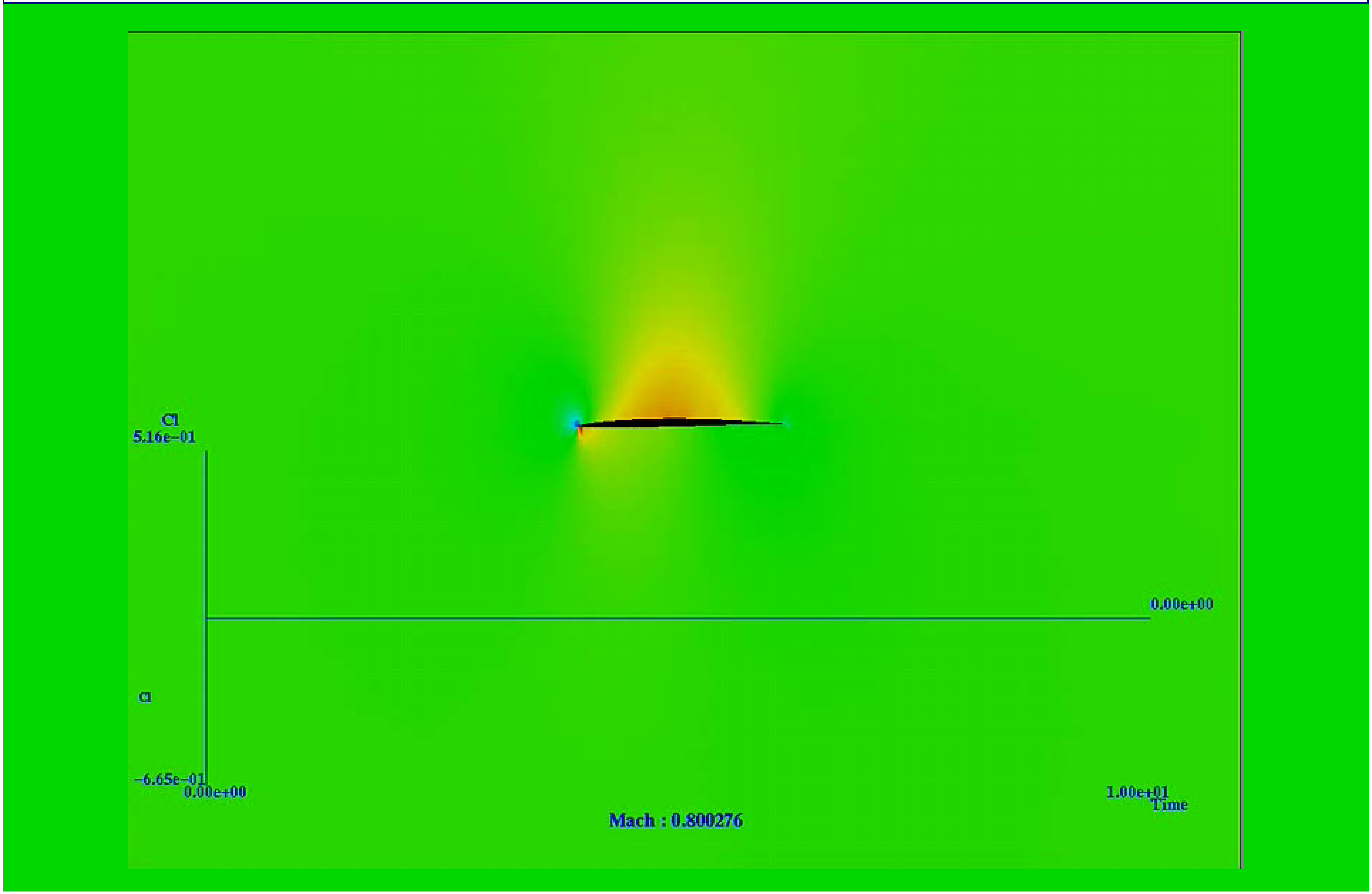
Colorado USA
Time : 0.002500

AEROELASTIC TRANSIENT RESPONSE

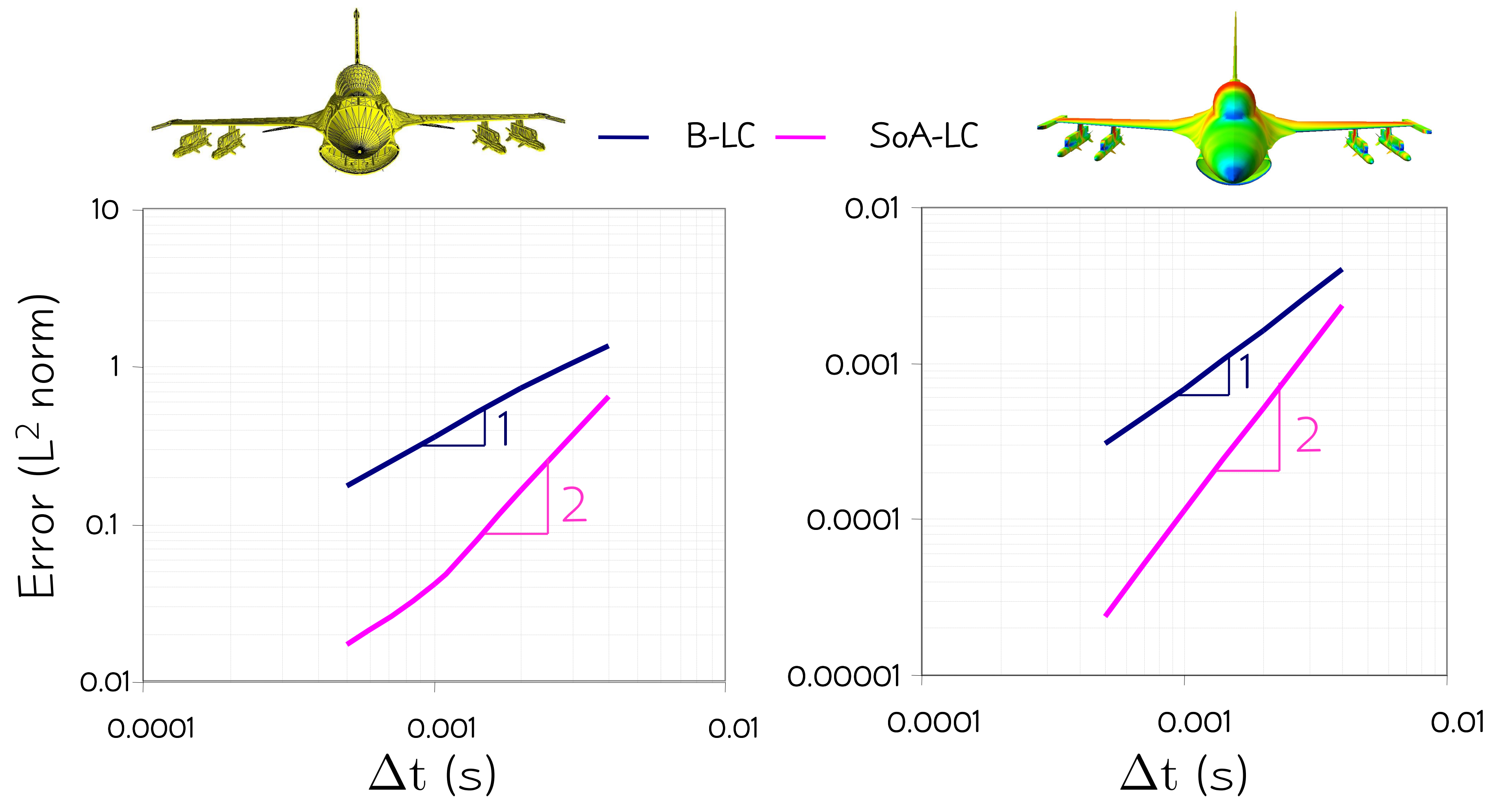
STANFORD MECHANICAL ENGINEERING



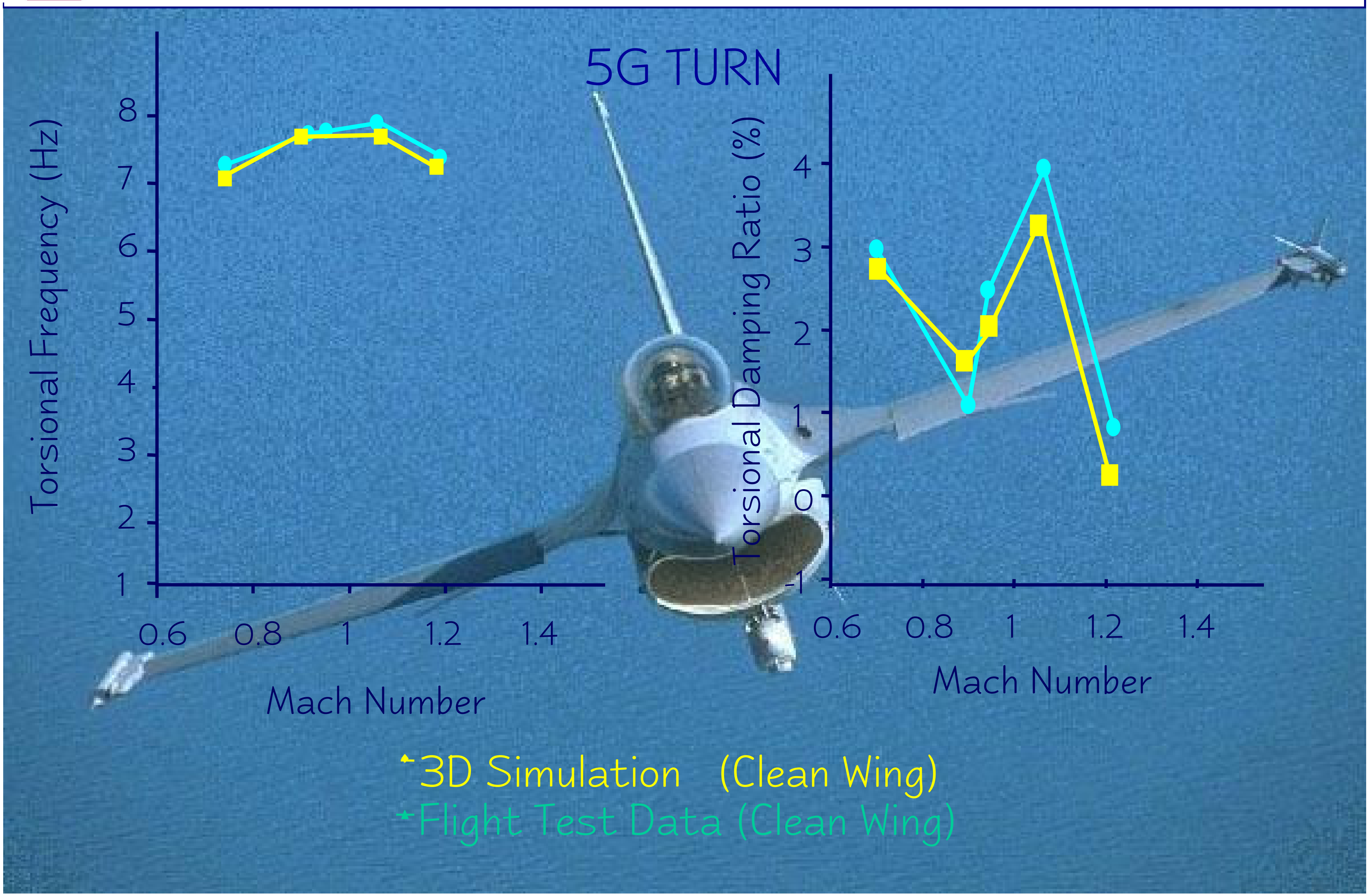
ACCELERATED FLIGHT



ACCURACY ON MOVING GRIDS



SAMPLE VALIDATION



ISSUES?

- The ART part is often forgotten/dropped by practitioners
  - loss of numerical stability and accuracy
  - “bad” reputation
- LeTallec (2001)
  - model incompressible flow problem
  - added-mass form of the governing equations
  - most primitive loosely-coupled time-integrator
  - asymptotic stability when $\rho_S << \rho_F$ & $\Delta t \rightarrow 0$
- Mok, Wall & Ram (2001)
  - low-speed flows, lightweight (shell) structures
  - weak instabilities observed when $M_S << K_S$

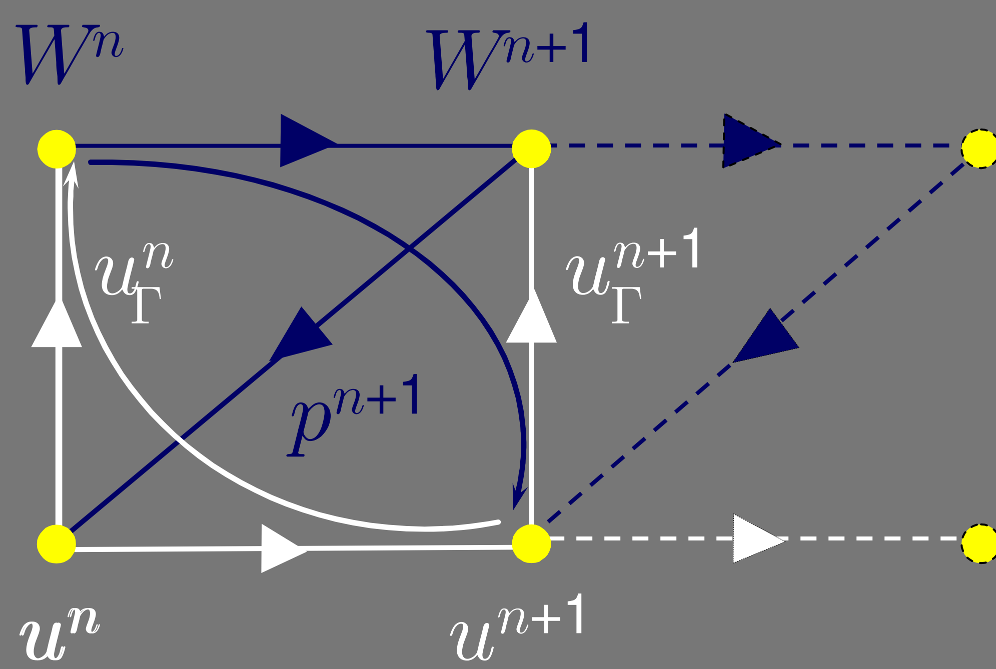
STRONGLY-COUPLED VERSION

- Dirichlet-Neumann inner-iterations (a la Quarteroni)

Dyn. Mesh $\quad x^n \qquad x^{n+1} = x^n + T^n \Delta u_\Gamma^n$

Fluid

Structure

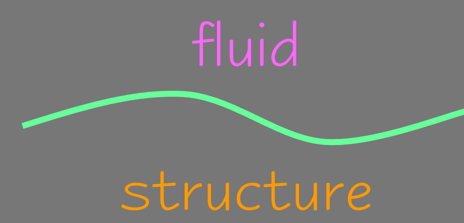


➡ relatively expensive proposition, unless necessary

LINK BETWEEN BOTH APPROACHES

- Piperno & Farhat (2001)

  - at convergence, inner-iterations conserve the energy transferred at the fluid/structure interface



$$\delta E^{n+1} = p^T{}_F \, (\delta u^{n+1} - \delta x^{n+1}) = 0$$

  - parameterized loosely-coupled partitioned schemes achieve the above property in an asymptotic sense

$$\delta E^{n+1} = p^T{}_F \, (\delta u^{n+1P} - \delta x^{n+1}) = O(\Delta t^q)$$

RECENT CONTROVERSY

- Causin, Gerbeau, and Nobile (2004)
  - blood flow in large human arteries
  - simplified model problem
  - most primitive loosely-coupled solver (no ART)
  - explicit structural time-integrator

$$\rho_F \frac{(v_F^{\,n} - v_F^{\,n-1})}{\Delta t} + \nabla p^n = 0$$

$$\mathrm{div}\, v_F^{\,n} = 0$$

$$\frac{(u_S^{\,n+1} - 2u_S^{\,n} + u_S^{\,n-1})}{\Delta t^2} + a\, u_S^{\,n} = p_\Gamma^{\,n}$$

RECENT CONTROVERSY

- Decomposition along the eigenvectors of the added mass operator leads to the following characteristic polynomial

$$\rho_S h \frac{(u_{Si}^{n+1} - 2u_{Si}^{n} + u_{Si}^{n-1})}{\Delta t^2} + \rho_F \mu_i \frac{(u_{Si}^{n} - 2u_{Si}^{n-1} + u_{Si}^{n-2})}{\Delta t^2} + a u_{Si}^{n} = 0$$

- Unconditional unstability when $\frac{\rho_S}{\rho_F} < \frac{\mu_{max}}{h}$

light — slender

- Poor convergence rate of Dirichlet-Neumann iterations under same condition

ENERGY ANALYSIS

- Incompressible flows

B-LC $\Delta E = -2\, c\omega\Delta t;\ c < 0$ $\rightarrow \Delta E > 0$

$$u^{n+1P} = u^n + 1\, \Delta t\, u^n + (1/2)\, \Delta t\, (u^n - u^{n-1})$$

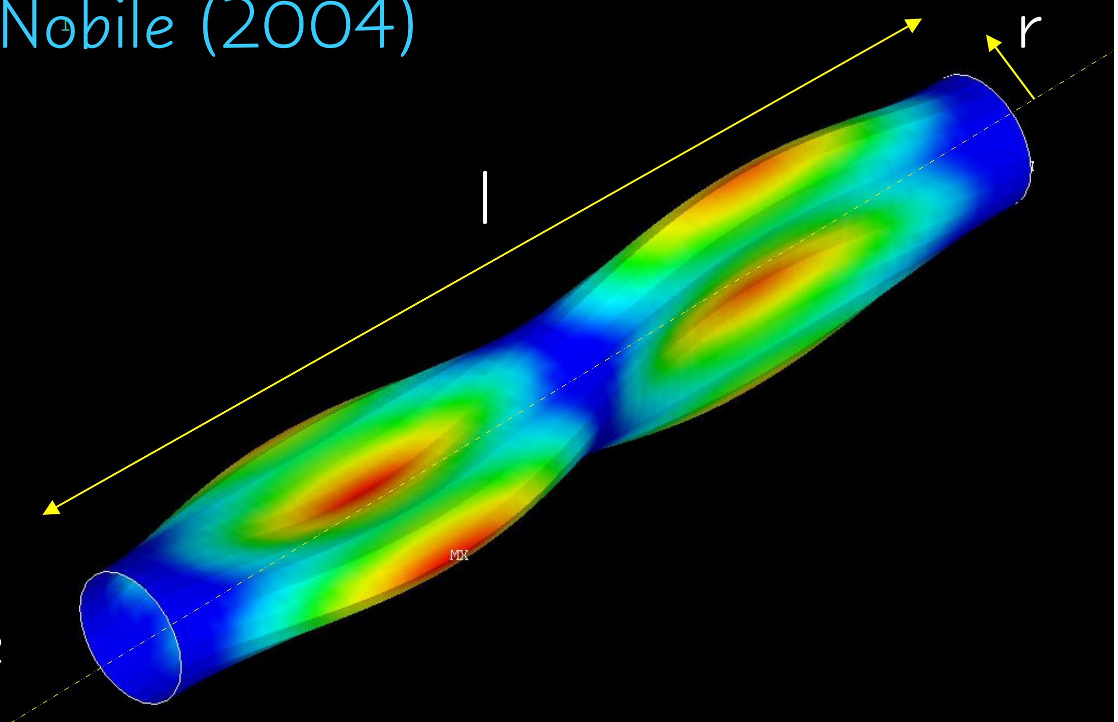
$$p^{n+1R} = 2p^{n+1} - p^n$$

SoA-LC $\Delta E = (17/24)\, c\omega^3\, \Delta t^3$ $\rightarrow \Delta E < 0$

➡ incompressibility is the source of numerical difficulties

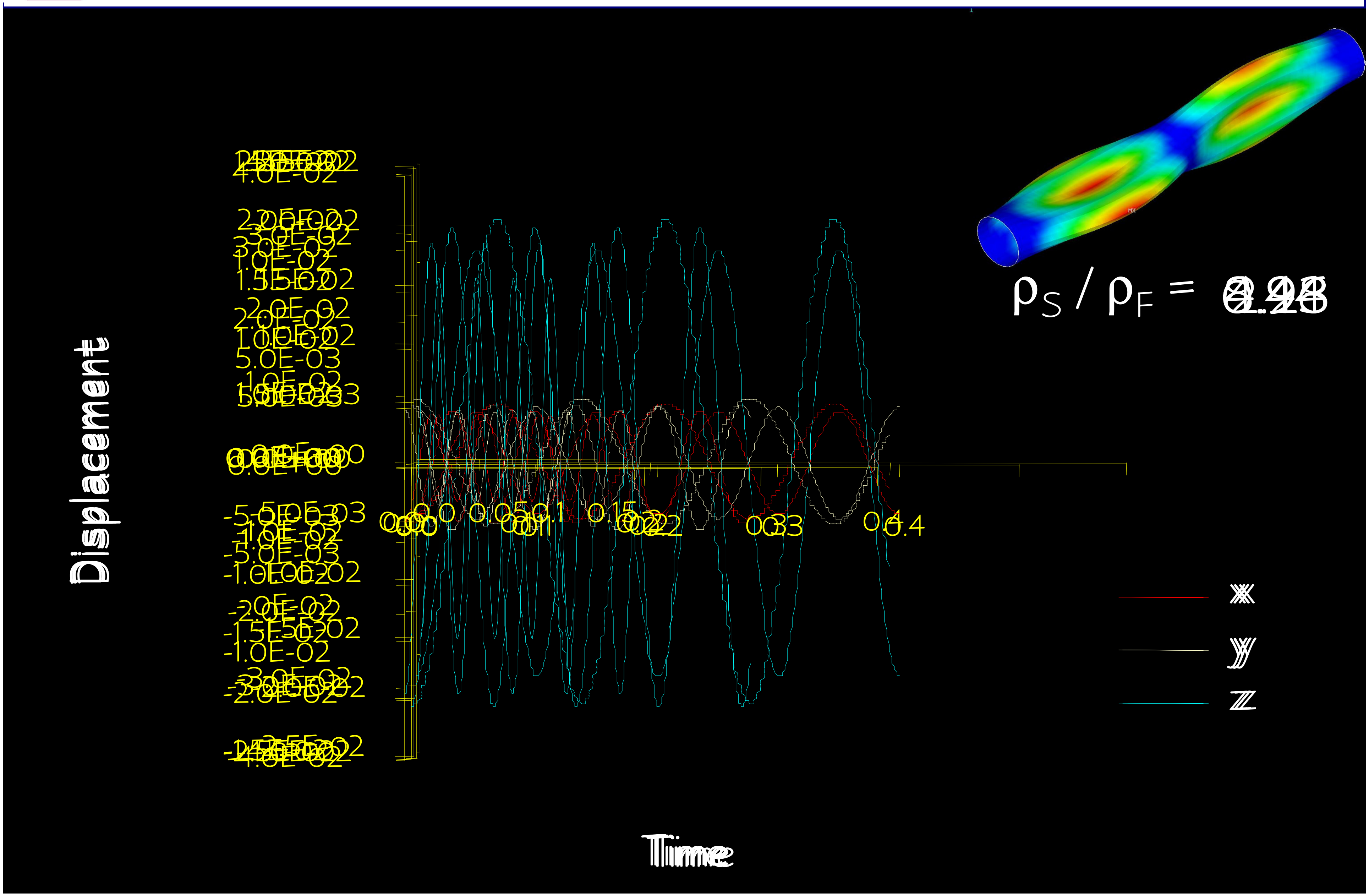
LOW-SPEED INTERNAL FLOW

- Causin, Gerbeau and Nobile (2004)
  - l = 6.0 cm
  - r = 0.5 cm
  - h = 0.1 cm
  - $\rho_F = 1.12\ g/cm^3$
  - v = 3.2 m/s
  - initial excitation by an eigen mode

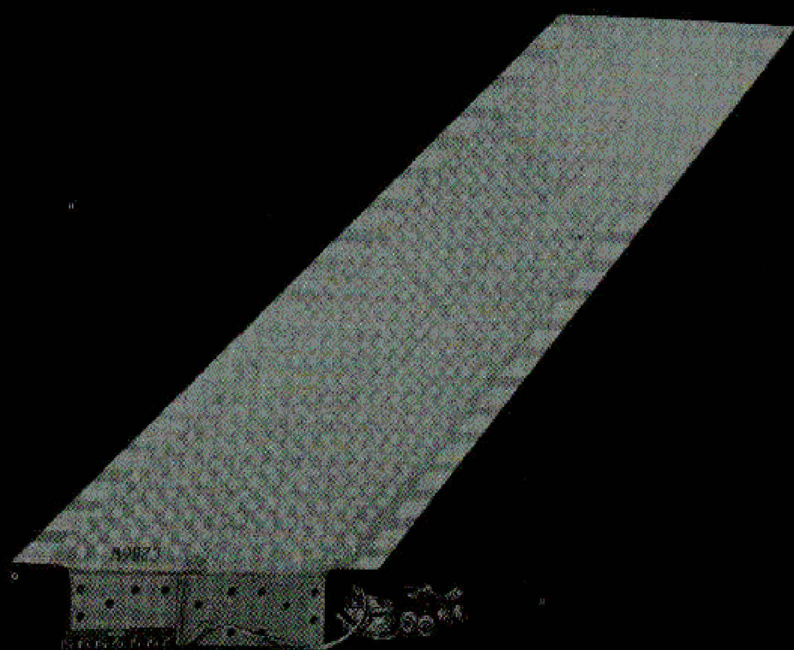


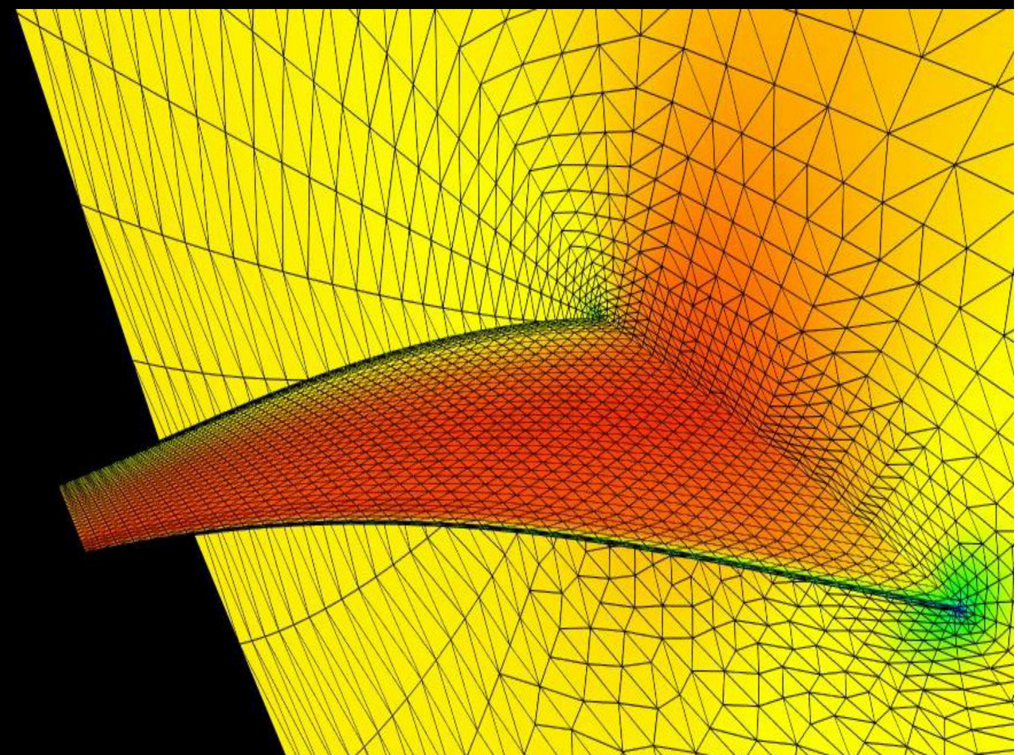
- Output: displacement field at node 1236
- $\rho_S < \rho_F$
- SoA LC with compressible flow solver

$\rho_S < \rho_F$



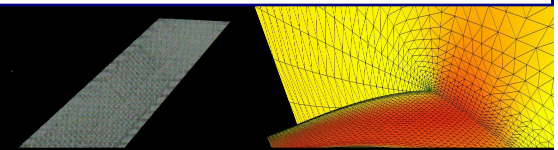
THE AGARD WING 445.6

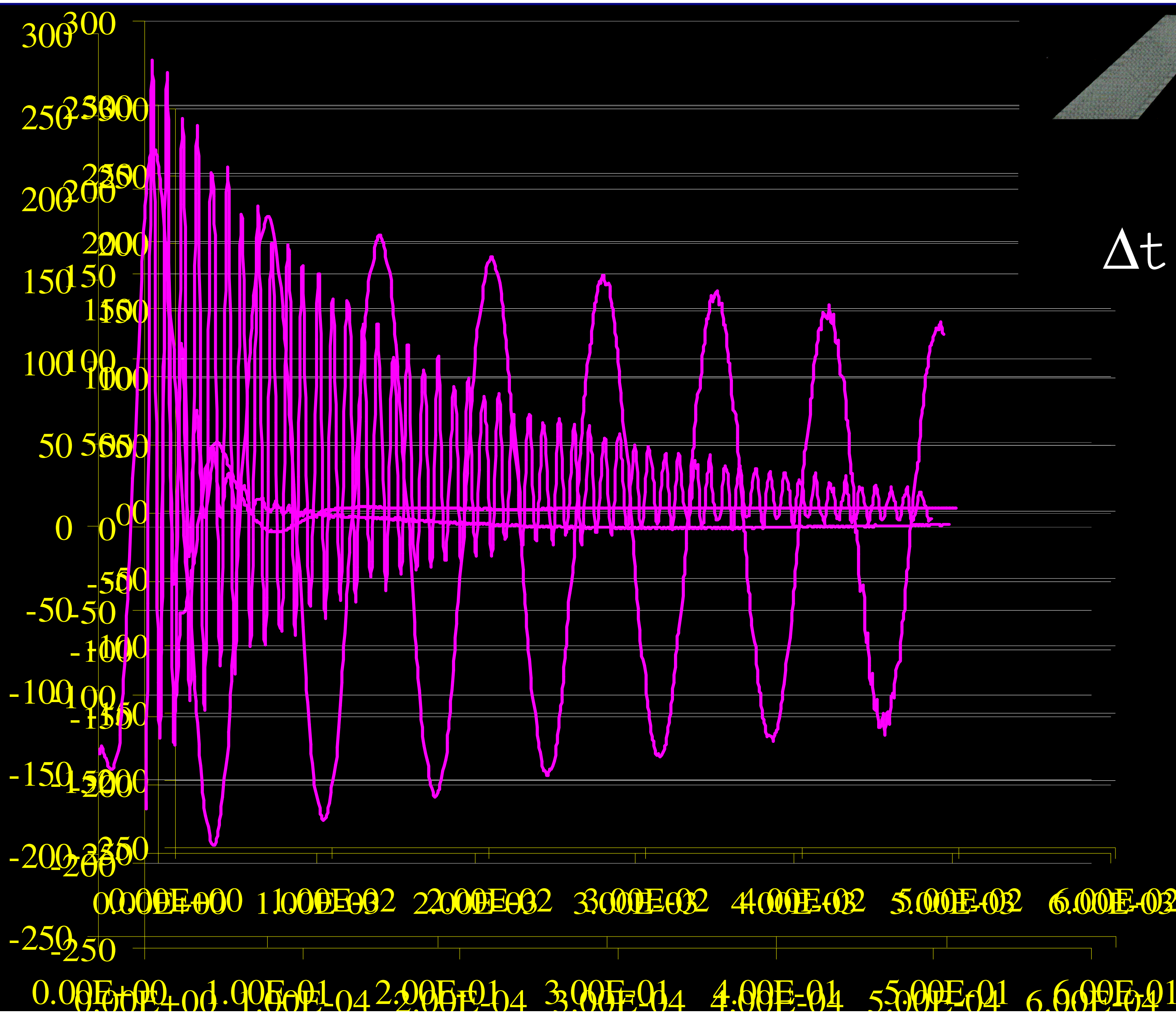




- $M_S << K_S$
- SoA LC with compressible flow solver

$M_S << K_S$





$\Delta t = T_2 / 30$

CONCLUSIONS

STANFORD MECHANICAL ENGINEERING

- GCL
  - DGCL and not GCL
  - in general, not related to accuracy but is a sufficient condition for consistency
  - related to ***nonlinear stability*** : at least for the nonlinear scalar conservation law, it is a necessary and sufficient condition for nonlinear stability

CONCLUSIONS

- Nonlinear stability and time-accuracy
  - nonlinear stability of coupled fluid/structure algorithm hinges on nonlinear stability of CFD scheme on *moving grids*
  - time-accuracy of coupled fluid/structure algorithm hinges on time-accuracy of CFD scheme on *moving grids*

CONCLUSIONS

- Loosely coupled solution algorithms for Class III problems
  - when smartly designed, they are VERY effective for *transient (unsteady) compressible* problems
  - smart design = parameterized design for control of accuracy and energy transfer at fluid/structure interface
  - not necessarily the most effective algorithms for steady-state problems
  - can suffer for incompressible fluid/structure interaction problems

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