On Penalization of Irreversibility in Variational Phase-Field Models of Brittle Fracture

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ABSTRACT

Irreversible evolution is one of the central concepts as well as implementation challenges of both the variational approach to fracture by Francfort and Marigo [1] and its regularized counterpart by Bourdin et al. [2, 3, 4] commonly referred to as a phase-field model of brittle fracture.

Irreversibility of the crack phase-field imposed to prevent fracture healing leads to a constrained minimization problem, whose optimality condition is given by a variational *inequality*. In our study, the irreversibility is handled via penalization. Provided the penalty constant is well-tuned, the penalized formulation is a good approximation to the original one, with the advantage that the induced *equality*-based weak problem enables a much simpler algorithmic treatment. We propose an analytical procedure for deriving the *optimal* penalty constant, more precisely, its *lower bound*, which guarantees a sufficiently accurate enforcement of the crack phase-field irreversibility. Our main tool is the notion of the optimal phase-field profile, as well as the Γ-convergence result. It is shown that the explicit lower bound is a function of two formulation parameters (the fracture toughness and the regularization length scale) but is independent on the problem setup (geometry, boundary conditions etc.) and the formulation ingredients (degradation function, tension-compression split etc.).

The optimally-penalized formulation is tested for two benchmark problems, including one with available analytical solution. We also compare our results with those obtained by using the alternative irreversibility technique based on the notion of the history field by Miehe et al. [5].

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