# Mode separation in a mixed-mode I/II problem by the J-integral and higher-order theories

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## ABSTRACT

This presentation deals with those kind of mechanical models which are applicable to model and describe laminated composite beams with interlaminar fracture under mode-I and mode-II fracture conditions.

In order to capture the complex fracture mechanical behaviour of delaminated composite beams first-, second- and third-order laminated beam models are presented with transversely isotropic and orthotropic material layers [1]. Based on the concept of four equivalent single layers and the system of exact kinematic conditions new types of model are developed and compared to each other [2]. Using the proposed displacement field the equilibrium equations are derived for the delaminated and undelaminated regions of the beam using the virtual work principle. Solving the developed system of ODEs for delaminated beams and utilizing the obtained more accurate displacement and stress field around the crack tip the application of the J-integral for semi-layerwise analysis are also introduced in order to determine energy release rate. In addition, by using the basic properties of the J-integral [3], novel and more exact mode-I/II partitioning is presented, as well, which can be considered an important contribution to the linear elastic fracture mechanics.

Moreover, the most important aim of this study is to investigate the mode mixity of several cases for composite beams. The effect of the delamination position through the thickness direction of the beam width with different material layers are examined based on the newly developed mechanical models. Finally, in terms of the energy release rates, the presented solutions of the evaluation techniques are compared to previously published models in the literature of linear fracture mechanics [4, 5, 6].

## REFERENCES

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