Nonlocal models for computing dynamic brittle fracture - CFRAC 2019

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ABSTRACT

Recent research activity has spawned a new class of nonlocal continuum models that posses a unique capacity to recover qualitative features of fracture without the a-priori prescription of crack paths. These continuum models introduced under the name peridynamics were proposed earlier in this century in the work of Silling [5]. The thrust of recent research by our group is to explore and develop methods for the efficient modeling of the fracture process with multiple interacting cracks. The nonlocal methods can be thought of as regularized models for fracture simulation. In previous work we show existence of nonlocal evolutions given by displacements taking values in a Hölder or Sobolev space at each instant in time. Here the length scale of nonlocality is ε and the solutions are shown to converge to a discontinuous fracture evolution with sharp jumps in the displacement as $\varepsilon \to 0$, see [1], and [2]. We fix ε, and present both finite difference and finite element methods for nonlocal fracture analysis. We show rigorous a-priori convergence rates on the relative error, derived in [2], [3]. For a time step Δt and uniform mesh size h the finite difference approximation converges to the actual Lipschitz continuous solution in the mean square norm at the rate $C_t\Delta t + C_s h/\varepsilon^2$ where ε. Here C_t and C_s are independent of Δt and h. Similarly for more regular solutions we have that the finite element approximation converges to a H^2 solution in mean square at the improved rate $C_t\Delta t + C_s h^2/\varepsilon^2$. In these models the crack set is modeled by sets for which the strain (as measured by a difference quotient) exceeds a critical value for which the force between two points begins to decrease for increasing strain, i.e., the material becomes weaker past a critical strain. These sets are theoretically shown to have volume proportional to ε, see [1]. Intuitively these sets can be thought of as fat cracks of finite length of width ε. We present simulations illustrating these features.

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