A Fractional Calculus Framework for Modeling and Prediction of Failure in Anomalous Materials

Eduardo A. B. de Moraes†, Jorge Suzuki† and Mohsen Zayernouri†*

* Department of Mechanical Engineering
Michigan State University
474 S Shaw Ln, East Lansing, MI 48824, United States
e-mail: zayern@msu.edu,

† Department of Computational Mathematics, Science and Engineering (CMSE)
Michigan State University
474 S Shaw Ln, East Lansing, MI 48824, United States

ABSTRACT

We develop a fractional calculus and probabilistic framework to model and simulate the failure of visco-elastic and visco-elasto-plastic materials. The employed models utilize Scott-Blair rheological elements of fractional order $0 < \alpha < 1$, which allows a constitutive interpolation between standard Hookean springs $(\alpha \to 0)$ and Newtonian dashpots $(\alpha \to 1)$, and introduces power-law stress-strain behaviors in both visco-elastic and visco-plastic regimes. We develop proper numerical methods for long and accurate time-integration through the efficient computation of the history load and the use of fractional return-mapping algorithms [1]. The damage and fatigue are incorporated through a phase-field model, in which the damage phase-field is a continuous dynamical variable, the fatigue is treated as a continuous internal variable [2], and many parameters are arbitrary or not physically measurable, treated here as random variables with a probability distribution. We employ the Monte Carlo and Probabilistic Collocation methods in the forward parameter uncertainty propagation and sensitivity analysis. The underlying parameters with highest uncertainty and sensitivity are closely related to terms in the free-energy potential that disregard nonlocal effects, which motivates the employment of the fractional-order operators to capture the intermittent and self-similar phenomena experimentally observed in fracturing materials.

REFERENCES

[1] Suzuki, J.L., Zayernouri, M., Bittencourt, M.L., Karniadakis, G.E., "Fractional-Order Uniaxial Visco-Elasto-Plastic Models for Structural Analysis", *Computer Methods in Applied Mechanics and Engineering*, 308, pp. 443−467, (2016).

[2] Boldrini, J. L., de Moraes, E. B., Chiarelli, L. R., Fumes, F. G., & Bittencourt, M. L. (2016). A non-isothermal thermodynamically consistent phase field framework for structural damage and fatigue. *Computer Methods in Applied Mechanics and Engineering*, *312*, 395-427