

# Dynamic fracture of brittle shells in an adaptive phase field framework based on LR NURBS

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## ABSTRACT

Phase field formulations of fracture have gained popularity within the last years since they do not require any ad hoc criteria for the prediction of fracture. Recently, several models investigating brittle and ductile fracture in shell structures have been published. The present work focuses on thin shells which are described within a convected coordinate system and whose kinematics follow from Kirchhoff-Love theory [2]. There are no rotational degrees of freedom but the surface deformation is sufficient to describe in-plane stretching and out-of-plane bending. In contrast to Ginzburg-Landau type of phase field evolution equations, we derive the governing equation for phase field evolution based on Griffith's theory of brittle fracture and minimization of the Helmholtz free energy. In these models, the stored elastic energy induces a fracture field. A suitable split has to be established which splits this energy into fracture-contributing and non-contributing terms. Given an elastic energy which is already split into a membrane and bending term within a large deformation setting, an energy split is derived which handles these contributions separately. A thickness integration is established for the bending term [3]. We adopt a higher order phase field model [1] which leads to a coupled system of two partial differential equations of fourth order. The required  $C^1$ -continuity is obtained by using an isogeometric analysis framework. An efficient implementation is established by using an adaptive formulation in space and time. Adaptive local refinement along the predicted crack path based on bivariate quadratic LR NURBS [4] is used. Those employ a local representation of the parameter space to dissolve the tensor-product mesh structure in standard IGA. A generalized- $\alpha$  scheme with adaptive time stepping is employed for the temporal discretization [5]. The resulting coupled system is fully linearized and solved within a monolithic Newton-Raphson scheme. Numerical examples are presented to illustrate crack propagation on deforming curved surfaces.

## REFERENCES

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