A Phase-Field Model in Incompressible Solids - Numerical Application and Error Estimation

Katrin Mang[*], Mirjam Walloth[†], Thomas Wick[*], Winnifried Wollner[†]

[*] Leibniz Universität Hannover
Institute of Applied Mathematics
Welfengarten 1, 30167 Hannover, Germany

[†]Department of Mathematics
Technische Universität Darmstadt
Dolivostrasse 15, 64293 Darmstadt, Germany

ABSTRACT

Within this talk, fractures in incompressible materials are simulated by using a variational phase-field approach [2]. The quasi-static phase-field fracture model in its standard formulation fails due to volume-locking effects if the observed solid is (nearly) incompressible [4]. To simulate crack growth in incompressible materials, the standard model is extended. The approach builds on a mixed form of the solid displacement equation resulting in two unknowns: a displacement field and a hydro-static pressure variable, see e.g. [1]. The fracture path is described with a phase-field function defined as a smoothed indicator variable. The crack irreversibility constraint is handled with a Lagrange multiplier. The resulting phase-field model for fractures in incompressible materials is explained in [3].
Finally, to resolve especially the transition zone between the broken and unbroken material and the region of the crack tip, we are interested in error estimation to achieve adaptively refined meshes. Our refinement strategy is based on an a posteriori error estimator developed for the variational inequality of the phase-field fracture model, see [5]. The numerical results of different mechanical tests are proposed to approve the quality of the new formulated model and the refinement strategy. These numerical tests include spatial mesh refinement studies and variations in Poisson's ratio approaching the incompressible limit.

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