A stabilized mixed formulation for phase-field modeling of fracture in porous media

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ABSTRACT

In the numerical approximation of a phase-field model of fracture in porous media with the Finite Element Method (FEM), the problem of the numerical locking due to high volumetric stiffness of the medium can occur. The causes of this state of incompressibility can be traced both to the hydraulic and the mechanical properties of the material. Regarding the hydraulic properties, it is well known that soils characterized by a low value of permeability show a local incompressible behaviour immediately after the application of a load. This causes oscillations of the numerical solution for the water pressure field when the inf-sup condition for the finite element interpolation functions is not fulfilled. Regarding the mechanical properties, when cracks are modeled with a phase-field approach based on a decomposition of the elastic energy into volumetric and deviatoric parts, the deviatoric stiffness becomes several order of magnitudes smaller than the volumetric one, causing an enlargement of the localization band of the phase-field variable and a dissipative residual force in the crack. To solve these problems, a mixed formulation for the mechanical part fulfilling the inf-sup condition for the choice of the interpolation functions has been developed.

In this work, we first present a mixed $\boldsymbol{u}$-p-d (displacement-pressure-phase field) finite element formulation for the phase-field modeling of fracture in single phase elastic materials. Then, this approach is extended to variably saturated porous media, developing a $\boldsymbol{u}$-p-p^w-d formulation (with p^w the pore liquid water pressure). Both models are shown to be stable. However, the higher degree of interpolation for the displacements field due to the inf-sup condition and the fine discretization required by the solution of the phase-field evolution equation lead to a very large number of degrees of freedom.

To reduce its computational cost a lower order of interpolation is proposed and a stabilization technique based on polynomial pressure projections is applied. This stabilization stems directly from the analogy between the incompressible Stokes equations [1] and the mixed formulation for incompressible elasticity, and has two interesting properties: the stabilization term is local, requiring only a few additional lines in the finite element code, and its influence is controlled by a material parameter.

Four numerical examples obtained using several standard (unstable) finite elements, mixed (stable) finite elements and low order stabilized elements are provided. They are: a one-dimensional tension test of an elastic bar; a two-dimensional shear test in single phase material and in water saturated material, and the study of the development of cracks in a clay domain during a desiccation process[2].

REFERENCES

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