On the Stability of an Evaporating Liquid Surface

Rolf Krahl*, **Eberhard Bänsch**†

* Leibniz Institute for Crystal Growth (IKZ)
Max-Born-Str. 2, 12489 Berlin, Germany
e-mail: krahl@ikz-berlin.de, web page: http://www.ikz-berlin.de/

†Friedrich–Alexander University of Erlangen, Chair of Applied Mathematics III
Haberstr. 2, 91058 Erlangen, Germany
e-mail: baensch@am.uni-erlangen.de, Web page: http://www.am.uni-erlangen.de/am3/

ABSTRACT

The stability of a free liquid surface was first studied by Yih [1] for an isothermal liquid film flowing down an inclined plane. Burelbach et al. [2] analyzed the stability of evaporating and condensing liquid films in the long-wave regime, where the wave length of the perturbation was assumed to be much larger than the thickness of the liquid layer. This scaling leads to the lubrication approximation. A comprehensive overview of this method is given by Oron et al. [3]. Much work has been devoted to analyzing the stability of films using the lubrication approximation. In contrast, significantly fewer publications exist on the stability of the evaporating surface of a liquid *volume*, where the lubrication approximation is not valid.

We consider a liquid volume and a gas phase without inert gas. The destabilizing effect on the free surface caused by the loss of mass due to evaporation is analyzed. The mechanism of this instability can be sketched as follows: the evaporation rate at the surface is proportional to the gradient of temperature in the normal direction. If the free surface is slightly perturbed, i.e. wavy, this gradient gets steeper in the troughs and less pronounced at the crests. This in turn results in a higher (lower) evaporation rate in the troughs (at the crests), which further increases the amplitude of the perturbation.

A linear stability analysis leads to the Orr–Sommerfeld equation for the stream function and a second order differential equation for the temperature. This system is solved semi-analytically. It is found that the loss of mass caused by evaporation has a destabilizing effect on the liquid surface. We investigate the influence of the characterizing parameters of the system, the Reynolds number Re, the Weber number We, the Prandtl number Pr, and the capillary number $Ca = We/Re$, on the stability of a perturbation with the wave number α. It turns out that surface tension is stabilizing the surface, while viscosity has a destabilizing effect. The capillary number is found to be the most important parameter.

The analytical results are compared with direct numerical simulations of the full system where the growth of an initial perturbation is considered. An excellent agreement with the analytical results is obtained.

REFERENCES

[1] C.-S. Yih *Stability of liquid flow down an inclined plane*, Phys. Fluids **6**, 321-334, 1963.

[2] J. P. Burelbach, S. G. Bankoff, S. H. Davis *Non-linear stability of evaporating/condensing liquid films*, J. Fluid Mech. **195** 463-494 1988.

[3] A. Oron, S. H. Davis, and S. G. Bankoff. *Long-scale evolution of thin liquid films*, Rev. Mod. Phys. **69** 931-980, 1997.