# Energy-integral model for heated falling films

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## ABSTRACT

We present a set of model equations describing the nonlinear dynamics of a liquid film falling down a heated inclined planar wall in the gravity field. Using an energy integral method (EIM), recently applied to a problem of an isothermal falling film on an inclined planar surface [1], a set of three coupled partial differential evolution equations for the local flow rate, local depth of the liquid layer and surface temperature is derived, where thermocapillarity is taken into account via the energy equation and the appropriate boundary condition.

At first order in the small long-wave expansion parameter $\varepsilon$ (the ratio of the mean thickness of the film to the characteristic wavelength of the disturbance in the flow direction), this model is found to be identical to the weighted-residual integral boundary-layer one, widely used in heated falling film problems [2-4]. At second order in $\varepsilon$, our EIM model becomes much simpler than the latter [3].

The linear stability of the Nusselt film flow is analyzed in the framework of our model and its results are compared to those of the Orr-Sommerfeld eigenvalue problem of the full Navier-Stokes / energy equations. Although there is not a significant difference between the first-order and second-order models in terms of the growth rate, the wave celerity of the second-order model is closer to the solution of the Orr-Sommerfeld analysis than that of the first-order model.

We investigate traveling wave solutions of our model by introducing the frame of reference moving with the steady wave speed. The resulting dynamical system is solved using the continuation software AUTO-07p [5]. We observe that a reconnection and an exchange of solution types between the primary and secondary branches of traveling wave solutions occur at certain values of the Reynolds and Marangoni numbers, which vary between the first- and second- order models.

This research was partially supported by the European Union via the FP7 Marie Curie scheme [PITN-GA-2008-214919 (MULTIFLOW)]. The authors thank Ms. E. Novbari for providing us with her code of AUTO software and with a tutorial of AUTO written by Dr. B. Scheid.

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