Modeling the transition to turbulence in pipe flow

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ABSTRACT

The transition to turbulence in pipe flow has been the subject of study for over 125 years [1]. There are at least two features of the problem that make it fascinating, but also difficult to analyze. The first is that when turbulence appears, it appears abruptly, and not through a sequence of transitions each increasing the dynamical complexity of the flow. Turbulence is triggered by finite-sized disturbances to linearly stable laminar flow. This hystertic, or subcritical, aspect of the problem limits the applicability of linear and weakly nonlinear theories. The second complicating feature is the intermittent form turbulence takes in the transitional regime near the minimum Reynolds number for which turbulence is observed. In sufficiently long pipes, localized patches of turbulence may persist for extremely long times before abruptly reverting to laminar flow [2]. In other cases, turbulent patches may spread by contaminating nearby laminar flow [3]. Both processes are effectively probabilistic and can only be adequately addressed through statistical analyses of many realizations. While minimal models have been useful in understanding generic features of spatiotemporal intermittency [4] until now there has been no model that captures the richness of behavior throughout the transitional regime of pipe flow.

Here a one-dimensional model is presented for transitional pipe flow. The model has two variables corresponding to turbulence intensity and axial center-line velocity. These evolve according to simple equations based on known properties of transitional turbulence. The model captures remarkably well the character of turbulent pipe flow and contains all of the following features: localized puffs with exponentially distributed lifetimes, puff splitting with exponentially distributed splitting times, slugs, localized edge states, unstable periodic orbits, a continuous transition to sustained turbulence via spatiotemporal intermittency, and a subsequent increase in turbulence fraction towards uniform, featureless turbulence.

References

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