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NONLINEAR DISTURBANCES OF THE VORTEX OCTAGON

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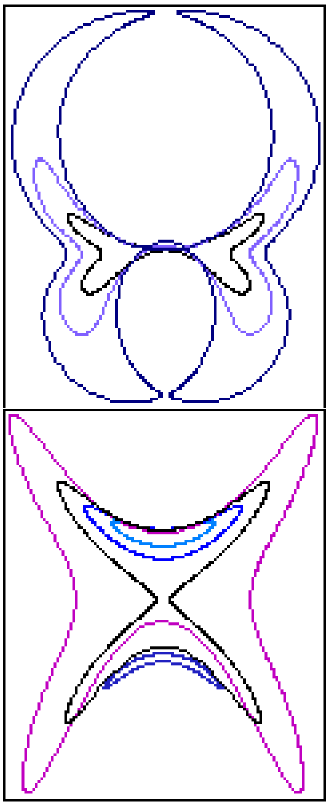
ABSTRACT

Competition between stohastic and regular properties in point vortices systems is in focus for many years because of connection of this problem and the model on turbulence. Point vortices arranged in a regular polygon performs the unstable system if number of vortices more than seven [1]. Exponential growth of the disturbances predicted by linear theory must lead to random chaotic behaviour. In this work the system of eight vortices is assumed to be couple interacting rings of four vortices. Hence the number of degrees of freedom is reduced to one. The governing equations of disturbances evolution



leads to autonomous second order ODE system [2]:

$$\frac{\mathrm{d}\varrho^2}{\mathrm{dt}} = \frac{4\Gamma}{\pi \mathrm{R}^2} \frac{\sin(4\varphi)}{\upsilon^2 + \upsilon^{-2} + 2\cos(4\varphi)}, \qquad \upsilon = 1 - 2/\varrho^2,$$

$$\frac{d\varphi}{dt} = \frac{\Gamma}{4\pi R^2} \frac{3(\upsilon^{-3} + \upsilon^2) + 11(\upsilon^{-2} + \upsilon) + 14(1 + \upsilon^{-1})\cos(4\varphi)}{\varrho^2(\upsilon^2 + \upsilon^{-2} + 2\cos(4\varphi))}.$$

This system describes the regular nonlinear behaviour of the vortex octagon disturbances, contributed to symmetry mentioned above. Unstable equlibrium state is $(\varrho = 1, \varphi = 0)$. It is shown that different types of the disturnances exist. The first one is closed curves round about the equlibrium point, the second type leaves it outside. The figures presents sample phase portraits of this types of solution. The choice of the solution branch depends on the initial value of disturbances. The criterion of solution separation is the value of φ_0 when the value of ϱ_0 is determined from expression:

$$\frac{2}{\varrho_0^2} = \frac{5}{3} + \frac{1}{3\sqrt{2}}\left(\psi \pm \sqrt{\psi^2 + 4\sqrt{2}\psi - 10}\right);\ \psi = \sqrt{29 - 21\cos(4\varphi_0)}.$$

The separation values of φ_0 is approximately equal to $\varphi_{cr} \approx 31\pi/736$. Solution of the first type correspond to $\varphi_{\mathrm{cr}} < \varphi_0 < \pi/4$, solution of the second type correspond to $\varphi_0 < \varphi_{\mathrm{cr}}$. Both sets of the solution is open and is not cross the equlibrium point.

REFERENCES

[1] T.H. Havelock *The Stability of Motion of Rectilinear Vortices in Ring Formation*, Phil. Mag. S. 7. **11(70)**, 617-633, 1931.

[2] E.G. Bord *On the Nonlinear Disturbances of Vortex Polygon*, Nonlinear Dynamics **3(2)**, 353-360, 2006.