Bifurcation analysis and transition to chaos in a large scale convection problem

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ABSTRACT

Many challenges in science and engineering are related to the complex dynamics of large scale dynamical systems. The identification of the invariant objects, such as fixed points, periodic orbits and homoclinic or heteroclinic orbits, and the stability characteristics of these objects are crucial issues in the understanding of the dynamics of any complex system. Efficient numerical Newton–Krylov methods for the continuation of periodic orbits and tori in high–dimensional dissipative dynamical systems have been recently reported. These matrix–free methods, which are based on the Arnoldi algorithm, also allow for the analysis of the stability of the corresponding invariant object along the continuation procedure. In particular, we adopted the Newton–Krylov method reported in [1] to extend the Galerkin spectral continuation method for steady solutions, described in [2] and used in [3], to the continuation of periodic orbits. Once the velocity and temperature fields are approximated by truncated expansions in terms of a divergence-free set of basis functions we obtain a system of dimension $O(10^4)$ whose unknowns are the coefficients in the expansions. We applied the developed methodology to determine the bifurcation diagram of steady and periodic solutions in the Rayleigh–Bénard problem inside a cube with adiabatc lateral walls and filled with air ($Pr = 0.71$). Previous knowledge of the bifurcation diagram of steady solutions, reported in [2], was used to identify the origin of several branches of periodic orbits that were continued with Ra. The method also allowed for the computation of those two–tori that are stable when they emerge at Neimark–Sacker bifurcations of periodic orbits. The symmetry properties of the problem, which is equivariant under the action of the sixteen elements in the symmetry group $D_{4h} = Z_2 D_4$, are responsible for the complexity of the bifurcation diagram and for the existence of various invariant symmetric subspaces in the space of solutions. In addition to the local bifurcations such as pitchfork, period doubling, fold and Neimark–Sacker bifurcations, global heteroclinic bifurcations have been found. Moreover, the analysis of different dynamical scenarios has allowed the identification of chaotic dynamics and the mechanisms leading to it.

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