Uncertainty Analysis for Flow of an Incompressible Fluid in a Sudden Expansion in Two-Dimensional Channel

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ABSTRACT

Numerical calculations of laminar flow in a two-dimensional channel with a sudden expansion exhibit a symmetry-breaking bifurcation at Reynolds number 40.45 when the expansion ratio is 3:1. In the experiments reported by Fearn, Mullin and Cliffe [1] there is a large perturbation to this bifurcation and the agreement with the numerical calculations is surprisingly poor. Possible reasons for this discrepancy are explored using modern techniques for uncertainty quantification.

When experimental equipment is constructed there are, inevitably, small manufacturing imperfections that can break the symmetry in the apparatus. In this work we considered a simple model for these imperfections. It was assumed that the inlet section of the channel was displaced by a small amount and that the centre line of the inlet section was not parallel to the centre line of the outlet section. Both imperfections were modelled as normal random variables with variance equal to the manufacturing tolerance. Thus the problem to be solved is the Navier-Stokes equations in a geometry with small random perturbations. A co-ordinate transformation technique was used to transform the problem to a fixed deterministic domain but with random coefficient appearing in the transformed Navier-Stokes equations. The resulting equations were solved using a stochastic collocation technique that took into account the fact that the problem has a discontinuity in parameter space arising from the bifurcation structure in the problem.

The numerical results are in the form of an approximation to a probability measure on the set of bifurcation diagrams. The experimental data of Fearn, Mullin and Cliffe are consistent with the computed solutions, so it appears that a satisfactory explanation for the large perturbation can be provided by manufacturing imperfections in the experimental apparatus.

The work demonstrates that modern methods for uncertainty quantification can be applied successfully to a bifurcation problem arising in fluid mechanics. It should be possible to apply similar techniques to a wide range of bifurcation problems in fluid mechanics in the future.

REFERENCES

[1] R M Fearn, T Mullin and K A Cliffe *Nonlinear flow phenomena in a symmetric sudden expansion*, J. Fluid Mech. **211**, 595-608, 1990.