

Inertial waves inside a liquid confined between two rotating coaxial cylinders: the case of small Ekman number.

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ABSTRACT

We investigate theoretically inertial waves inside a liquid confined between two co-rotating coaxial cylinder for small viscosity and high angular velocity (i.e. small Ekman numbers), a parameter range of interest for many geophysical applications. In this case inertial waves show multiple reflections at the walls before the waves are damped by diffusion. We allow for the inner cylinder surface to be vertical or slightly inclined (truncated cone).

The system is governed by a hyperbolic second order differential equation. For the case of vertical cylinder walls the eigenmodes can be easily calculated. Actually, in this case the hyperbolic boundary value problem seems to be as well posed as an elliptical boundary value problem. However, when the inner cylinder wall is inclined, the hyperbolicity leads to internal shear layers corresponding to singularities for the inviscid case. The geometrical structure of the shear layers can be explained by inertial waves, trapped on limit cycles. The shape of the limit cycle defines the structure of the internal shear layers [1,2,3]. In fact, the spectrum of regular modes, existing for the case of vertical cylinder walls, completely vanishes when the inner wall is inclined. The question remains if the "spectrum" of limit cycles for weakly inclined walls still captures some part of the eigenvalue spectrum of trajectories for the vertical walls.

To answer this question we plot the attractor frequency "spectrum" for a small cylinder inclination angle and we compare it with the eigenspectrum for the case of vertical cylinder walls. In this way we hope to understand better the asymptotic behavior of the problem for decreasing inclination angles [4].

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