# On the Early Evolution in the Transient Benard - Marangoni Problem

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**ABSTRACT**

The Benard - Marangoni mechanism coupling nonuniform surface - temperature distribution and convection across 'favourable' bulk - temperature gradients is of fundamental importance in hydrodynamic stability theory as well as considerable relevance to a wide variety of engineering (e.g. coating) applications and natural (e.g. physiological) phenomena. We here focus on the early evolution of small (linear) perturbations following the sudden (step - function) exposure of a liquid layer to a cold adjacent atmosphere. On a time scale short relative to that characterizing thermal relaxation across the liquid layer the temperature distribution is non linear and highly transient. Thus, the conduction reference state may not be regarded quasi steady. We accordingly consider thr initial – value problem (rather than the common eigenvalue - problem formulation) and obtain a Volterra - type integral equation governing the evolution of surface - temperature perturbations. Assuming an *O(1)* Biot number we study the effects on perturbations evolution of the wavenumber, the Prandtl number (*Pr*) and the effective Marangoni number ( *Ma*, which, from the dominant balance in the thermal perturbation equation, is based on the current values of the width of the thermal boundary layer adjacent to the liquid surface and the temperature difference across this layer, respectively). Explicit results are first presented in the limit of large *Pr>>1* wherein the hydrodynamic perturbation problem is quasi steady. The dominant perturbations correspond to wavenumbers and convection effectively confined to the thermal boundary layer. Typical of the results is the non – monotonical temporal evolution of perturbations which initially diminish and only after some finite delay time start to grow. These trends are rationalized in terms of the Marangoni mechanism by observing that while the magnitude of the reference - state temperature gradients remain essentially constant, they extend over a widening thermal boundary layer allowing for enhanced convective effects. Thus, since perturbations may be introduced at all positive time, those evolving on the favourable background of further developed (wider) thermal boundary layers may take over earlier introduced perturbations. This suggests the existence of a non - zero ‘optimal’ time of appearance of perturbations which will eventually dominate the instability process. We study the effects of finite *O(1)* values of *Pr* on the evolution of the hydrodynamic perturbation problem which is no longer quasi steady. Increasing *Pr* at a constant *Ma* is destabilizing owing to the reduced delay time of the dynamic response to variations of surface - temperature perturbations. This, in turn, enhances the coupling underlying the Benard - Marangoni mechanism. It is further demonstrated that with increasing wavenumber convergence to the asymptotic *Pr>>1* limit is practically achieved at smaller *Pr*.