On high order finite difference methods for global instability analysis

- BIFD2011 -

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ABSTRACT

The main goal of this work is to apply a new stable high order (order q) finite-difference method (FD-q), based on non-uniform grid points [1], to instability analysis of complex flows. Motivation is offered by the high cost of spectral methods typically used for this class of problems [2], which can be substantially reduced by exploiting the sparsity offered by the FD-q scheme, without sacrificing accuracy.

The new method has been validated on the classic Orr-Sommerfeld equation, comparing its results at different orders against spectral collocation method based on the Chebyshev Gauss-Lobbatto (CGL) points, and results obtained by alternative FD numerical methods, such as Dispersion-Relation-Preserving finite difference (DRP) [3] and Compact finite difference (also known as Padé schemes) [4]. Figure 1 shows the relative error of the different techniques in recovering the Plane Poiseuille Flow (PPF) eigenspectrum; it may be seen that at all orders of discretization the FD-q method presents better resolution properties, reaching convergence before any alternative FD method. In addition, beyond a certain modest value of discretization points, N, the FD-q method employs $O(Nq)$ discretization points, as opposed to the $O(N^2)$ points required by te CGL. This message is reinforced by results of the Blasius eigenspectrum: figure 2 shows that the FD-q method not only correctly obtains the leading part of the discrete eigenspectrum in this flow, but, when compared with the standard CGL approach, it also reproduces a more vertical line as the discrete representation of the continuous spectrum.

These favorable resolution properties of the FD-q method are subsequently applied to recover the BiGlobal eigenspectrum of square regularized lid-driven cavity at $Re = 1000, \beta = 15$. Table 1 shows convergence study results of three FD-q and a CGL discretization, which establish the FD-q spatial discretization as a viable alternative to the latter method. The three-dimensional reconstruction of the BiGlobal modes obtained by the FD-q method are shown in Figure 3. Results of application to other complex flows will be available at the time of the Conference.

REFERENCES

[1] Hermanns M. and Hernández J. A. *Stable high-order finite-difference methods based on non-uniform grid points distributions*, Int. J. for Num. Meth. in Fluids **56**, 233-255, 2008.

[2] Theofilis V. *Global linear instability*, Annu. Rev. Fluid Mech. **43**, 319-352, 2011.

[3] Tam C. and Webb J. *Dispersion-Relation-Preserving Finite Ddiference Schemes for Computational Acoustics*, J. Comput. Phys. **107**, 262-281, 1992.

[4] Lele S. K. *Compact Finite Difference Schemes with Spectral-like Resolution*, J. Comput. Phys. **103**, 16-42, 1992.

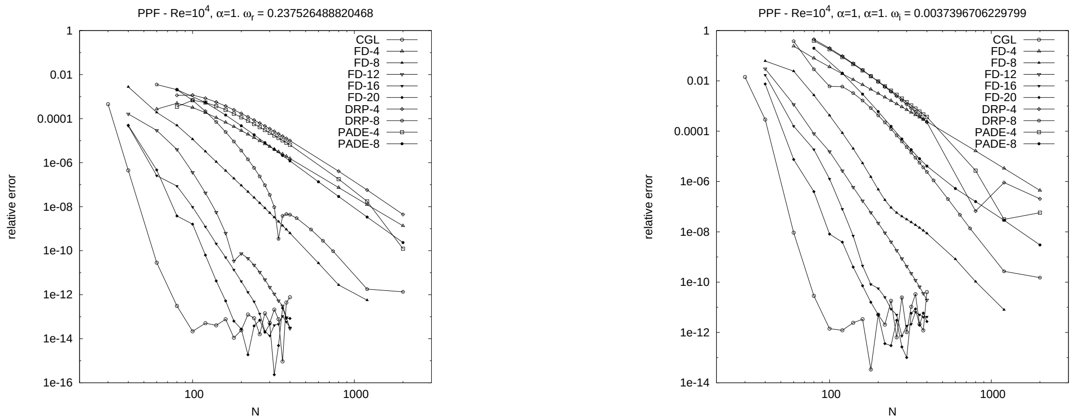

Figure 1: Relative error comparison for different numerical methods in a Plane Poiseuille flow

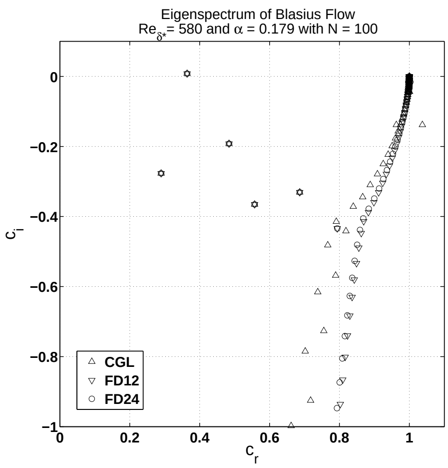

Figure 2: Eigenspectrum in a Blasius boundary layer flow

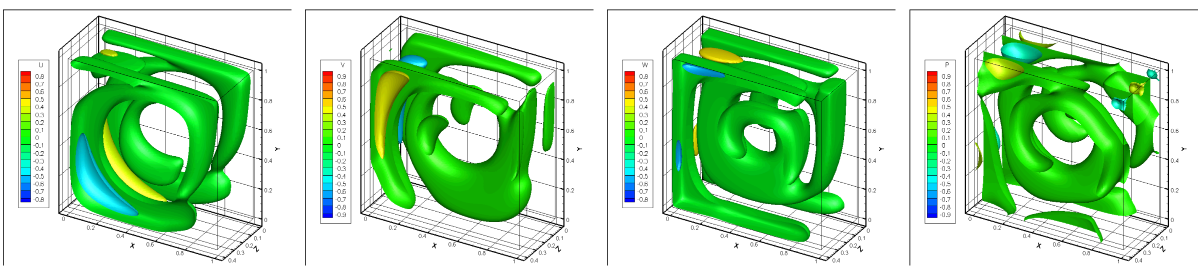

Figure 3: Amplitude functions corresponding to the most unstable stationary BiGlobal eigenmode (S1). Left-to-right: $\hat{u}(x, y, z), \hat{v}(x, y, z), \hat{w}(x, y, z), \hat{p}(x, y, z)$.

Table 1: BiGlobal instability analysis of the regularized lid-driven cavity flow at $A = 1$, $Re = 1000$ and $\beta = 15$. Convergence of the most unstable mode for different spatial schemes.

N	FD-8	FD-16	FD-24	CGL
32	0.1167494	0.1085822	0.1088816	0.1083455
40	0.1121854	0.1081735	0.1082842	0.1083697
48	0.1095856	0.1085076	0.1083438	0.1083468
56	0.1086749	0.1084422	0.1083214	0.1083409
64	0.1084124	0.1083639	0.1083276	0.1083386
72	0.1083568	0.1083268	0.1083310	0.1083383