Bifurcation analysis of salt induced convection in a porous square cavity

Daniel Henry*, **Ridha Touihri**†, **Rachida Bouhlila**†, **Hamda Ben Hadid**

Laboratoire de Mécanique des Fluides et d'Acoustique, CNRS/Université de Lyon
Ecole Centrale de Lyon/Université Lyon 1/INSA de Lyon
ECL, 36 avenue Guy de Collongue, 69134 Ecully Cedex, France
e-mail: daniel.henry@ec-lyon.fr, web page: http://lmfa.ec-lyon.fr

†LAMSIN et LMHE, ENIT
Campus Universitaire, Le Belvédère, BP 37, 1002 Tunis, Tunisie

ABSTRACT

The presence of salt influences the groundwater dynamics in a number of environmentally important problems such as saltwater intrusion in exploited coastal aquifers or aquifers overlying salt formations. In these situations, salt induced destabilizing density differences can produce convective currents through the gravitational forces [1]. A model configuration of these situations is a porous square cell filled with water, where convection is the result of a stronger salinity at the top. In this configuration, it is known that the onset of convection is at $Ra = 4\pi^2$ [2] and that different convective flows are found depending on the value of Ra and on the approximations of the model. In this work, we propose to study the dynamics of the convection in such a configuration with a simple Darcy model coupled with a salt mass fraction conservation equation. The problem is solved in the square domain using a spectral element method and a continuation method allowing steady state solving, stability analysis, and direct calculation of the different bifurcation points is used to map the dynamics of the flow.
The successive primary bifurcation points correspond to different flow structures. The first point which appears for $Ra = 4\pi^2$ corresponds to a one-roll structure. Two-roll and three-roll structures (along the horizontal) are then found, before a (1,2) structure (with two roll in the vertical), a four-roll structure and a (3,2) structure. If the one-roll branch which emerges from the first primary bifurcation point is stable at onset, the other branches are unstable. Three of them, however, will eventually be stabilized, the two-roll branch beyond a secondary bifurcation point, the three-roll branch beyond two secondary bifurcation points, and the four-roll branch after a more complex evolution. The two other branches will remain unstable. The bifurcation diagram of our system then has four different branches which become stable at successive Ra (between $Ra = 4\pi^2$ and $Ra = 264$) and are eventually destabilized at Hopf bifurcation points. These Hopf bifurcation points range from $Ra = 383$ to $Ra = 664$, giving different lengths for the stable branches and explaining why the four steady solutions can be obtained for $Ra = 300$, and only three of them for either $Ra = 200$ or $Ra = 500$. Note that a fifth branch corresponding to distorted four-roll solutions emerges from the four-roll branch at high Ra ($Ra = 581$). Only the Hopf bifurcation point on the one-roll branch is supercritical and gives rise to an oscillatory flow with very regular oscillations. The Hopf bifurcation points on the other branches are sub-critical and give rise to complex periodic flows or even to chaotic flows.

REFERENCES

[1] O. Kolditz, R. Ratke, H.-J. G. Diersch, W. Zielke *Coupled groundwater flow and transport: 1. Verification of variable density flow and transport models*, Advances in Water Resources **21(1)**, 21-46, 1998.

[2] A.E. Scheidegger *The physics of flow through porous media*, Third edition, University of Toronto Press, 1974.