Analytical estimates for stability boundaries and nature of the bifurcation in a simple thermoacoustic system

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ABSTRACT

Thermoacoustic instabilities mostly appear as self-sustained nonlinear limit cycle oscillations. Unacceptable levels of vibrations, structural damage and even flame flashback or blowout occur as the pressure oscillations grow to these limit cycles. It is therefore essential to identify the bifurcation behavior of the thermoacoustic system including the characteristics of the limit cycles such as amplitude and frequency content and their stability properties.

A simple low order model for thermoacoustic instability is derived for the horizontal Rijke tube [1]. In this model the nonlinear response of the heater is modeled based on a correlation for the heat release rate given by Heckl [2] using an explicit time delay term. The model thus obtained still retains the diverse dynamical behavior of the thermoacoustic instability in a Rijke tube. Subramanian *et al.* [3] used the method of numerical continuation to obtain the bifurcation behavior of this mathematical approximation of a horizontal Rijke tube. It was observed that when typical system parameters are varied, the trivial steady state of the Rijke tube loses stability through a subcritical Hopf bifurcation. The ensuing limit cycles are unstable and they coexist with the stable trivial steady state. The unstable periodic solutions undergo a fold bifurcation giving rise to stable periodic solutions. The region between the Hopf and the fold bifurcation represents the bistable region where the system either shows a stable trivial state or large amplitude self-sustained oscillations. The possibility of a bistable region can be ascertained by determining the nature of the Hopf bifurcation and the extent of the bistable region can be completely characterized by locating the Hopf and fold points. In this paper, we present an analytical study of the nature of the Hopf bifurcation and estimates of the linear (Hopf) and global (fold) stability boundaries.

The nature of the bifurcation along with estimates of the amplitude of the periodic states near the Hopf point and their stability are studied using the method of multiple scales. Our analysis reveals that only subcritical Hopf bifurcations occur in this model. Estimates for the Fold bifurcation points are obtained by employing the method of harmonic balance while the Hopf bifurcation points are obtained via the classical eigenvalue analysis. A very good agreement has been obtained between the results obtained from the analytical methods in this work with results from numerical continuation [3]. Analytical estimates of the global boundary serve as an important design tool since the system does not undergo any oscillations for parameter values beyond these boundaries irrespective of the strength of the disturbance.

REFERENCES

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