Numerical prediction of the properties of solitary waves in the drag-inertia regime

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ABSTRACT

Derivation of simplified models, based on long-wave expansions, for the description of liquid film flow has a long history marked by classical results such as the one-equation model of Benney and the two-equation model of Shkadov [1]. A crucial test of such models is the correct prediction of the properties (shape, maximum height, phase velocity) of solitary waves as a function of the distance from the instability threshold. The latter is usually quantified in terms of the reduced Reynolds number, $\delta = 3^{4/3} Re^{11/9} Ka^{-1/3}$, where the Reynolds number is defined in terms of the undisturbed film thickness and mean velocity, $Re = u_N h_N / \nu$, and the Kapitsa number, $Ka = \sigma / \rho \nu^{4/3} g^{1/3}$, contains only physical properties and compares capillary and viscous diffusive effects. Considering a vertical wall and assuming negligible streamwise viscous diffusion effects, i.e. large Kapitsa numbers, δ is the only independent parameter of the rescaled equations. It is recalled that, though most models predict similar behaviour close to the threshold (onset of drag-gravity regime), they exhibit large differences from each other at intermediate and large values of δ, i.e. when inertia becomes significant (transition region and drag-inertia regime) [2]. Thus, rigorous simulations emerge as the only means to resolve this issue.

The present work computes accurately the properties of stationary, traveling wave by solving the Navier-Stokes equation by a finite-element technique [3], implemented with periodic boundary conditions and strong mesh refinement in the vicinity of the solitary wave. Solitary-like waves are derived by considering a long enough computational domain. However, it is shown that lengths of the order of 10^4-10^5 times the film thickness are necessary for the properties of the wave to converge asymptotically to the true solitary limit with accuracy of less than 0.5%.

Both the phase velocity and the wave height exhibit inflection points in the transition region, then maxima at intermediate values of δ, and finally a drop to a plateau at high enough δ. These are unique characteristics of the full second-order model by Ruyer-Quil and Manneville [1], whereas all other models predict either a monotonic increase or a monotonic, asymptotic limit. In particular, simulations and the above second-order model agree quantitatively in the drag-gravity regime and the transition region to the drag-inertia regime, but only qualitatively in the drag-inertia regime. The behaviour deep in the drag-inertia regime is found to depend on Ka, which in the present simulations is varied in the range 200-10000. With increasing Ka, the maxima that occur at intermediate δ become steeper. However, whereas the high-δ limit of the phase velocity appears almost unaffected, that of the wave height increases roughly linearly with Ka, which underlines the stabilizing effect of viscous diffusion at low Kapitsa numbers.

Research partially supported by the EU/FP7 PEOPLE ITN "MULTIFLOW" (GA-2008-214919)

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