Can quasi-static drops climb uphill on an oscillating substrate?

- BIFD 2011 -

Eugene S. Benilov*

* Department of Mathematics, University of Limerick
Limerick, Ireland
e-mail: eugene.benilov@ul.ie, web page: http://www.staff.ul.ie/eugenebenilov/hpage

ABSTRACT

Everyday experience suggests that, in the absence of a towel, the best way to remove liquid drops from an object is to shake it, so the drops would slide down and, eventually, fall off. In a recent paper, however, Brunet, Eggers, and Deegan [1] have shown experimentally that, on a sloping plate vibrating vertically, drops can actually climb uphill.

An attempt to explain the effect of climbing drops was carried out by Benilov [2] who examined the problem using a quasi-static approximation (assuming the oscillations to be slow, so the drop's shape is determined by the balance of surface tension, gravity, and inertial force). It was argued that the drop can climb uphill provided the time dependence of the plate's acceleration, $a(t)$, involves narrow/deep 'troughs' and wide/low 'crests' (or rather 'plateaus'). However, this wasn't the case in the experiments of [1] where $a(t)$ was sinusoidal and, thus, had symmetric troughs and crests.

Another theoretical study of the results of [1] was carried out by Benilov and Billingham [3]. Not employing the quasi-static approximation, they assumed that the inertial force and gravity are weaker than surface tension, whereas the Reynolds number is large. Within the framework of such a model, drops were shown to indeed be able climb uphill for a sinusoidal $a(t)$.

Still, one discrepancy remained: in a significant proportion of the experiments reported in [1], the drops were indeed quasi-static, for which case [2] appears to predict that they cannot climb uphill (which disagrees with the experiments). Another shortcoming of both theoretical studies [2-3] is that they considered two-dimensional drops (where the real-life drops are, of course, three-dimensional.

The present work resolves the above discrepancy. It is shown that an asymptotic regime was missed in [2] where quasi-static drops can indeed climb uphill due to interplay between the inertial force and hydrostatic pressure. The results obtained are extended to three-dimensional drops. Finally, it is shown that, if $a(t)$ involves narrow/deep troughs and wide/low plateaus, the drop climbs uphill much faster than that in the case of sinusoidal $a(t)$.

REFERENCES

[1] P. Brunet, J. Eggers and R.D. Deegan, "Vibration-induced climbing of drops", *Phys. Rev. Lett.*, Vol. **99**, pp. 144501/1-4, (2007).

[2] E.S. Benilov, "Drops climbing uphill on a slowly oscillating substrate", *Phys. Rev. E*, Vol. **82**, 026320/1-8, (2010).

[3] E.S. Benilov and J. Billingham, "Drops climbing uphill on an oscillating substrate", *J. Fluid Mech.*, to appear, (2011).