

# Wavy film flow down a vertical plate: Comparisons between the integral approaches results and the full-scale computations

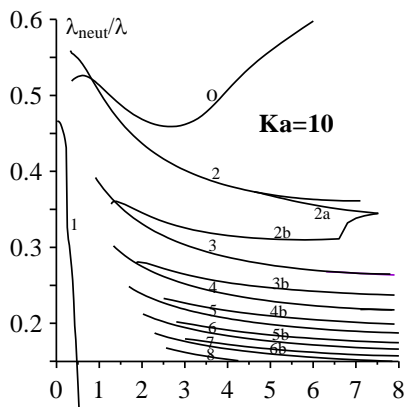
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## ABSTRACT

This paper is devoted to a theoretical analysis of nonlinear two-dimensional waves using the Navier-Stokes equations in their full statement. Steady-state travelling waves have been found and an analysis of their linear stability has been carried out using the Floquet theory (disturbances have the form  $\Psi(x, y)\exp[-\gamma t + 2\pi i Q x]$ ,  $Q \in [0, 1]$ ,  $\Psi(x+1, y) = \Psi(x, y)$ ,  $\gamma$  is the spectrum of the eigenvalues). We use also Shkadov's approach and "the regularized integral model" [1] to compute the traveling waves, to analyze their linear stability (in this case, disturbances have the form  $\Psi(x)\exp[-\gamma t + 2\pi i Q x]$ ) and to present the detailed comparison between the approaches. It is shown that the solutions obtained using the Navier-Stokes equations are qualitatively different from the solutions of Shkadov's integral approach starting from some values of the Kapitza number (Ka). The solutions of "the regularized integral model" are in excellent agreement with the Navier-Stokes calculations for  $\text{Re}/\text{Ka} \leq 2$ . It is found that the solutions of all models considered here have an internal



vortex at moderate Reynolds numbers. We carry out a detailed stability analysis of the waves with respect to disturbances of the same period ( $Q=0$ ) and calculate the bifurcation lines of the 1-st and 2-d family waves on the plane  $(\lambda/\lambda_{\text{neut}}, \text{Re}/\text{Ka})$  with respect to disturbances with  $Q=0$  at different values of the Kapitza number ( $\lambda$  is the wavelength of the nonlinear wave,  $\lambda_{\text{neut}}$  is the wavelength of the neutral disturbance). On line 0-1 the Landau-Hopf bifurcation takes place for the  $\gamma_1$ -family and on the lines 2b-6b for the  $\gamma_2$ -family waves. The real part of the corresponding eigenvalue goes through zero on lines 0-6b at nonzero value of the imaginary part. On lines 2-7, the real eigenvalue goes through zero and we have the "returning" lines here. These lines form a multi-fold and multi-sheet surface where the different types of

solutions can be computed using the continuation principle and starting from the small values of  $\text{Re}/\text{Ka}$ . The distance between the folders decreases with the wavelength decreasing and we have more and more different nonlinear solutions at one set of parameters. Most of the solutions are unstable with respect to disturbances of the same period ( $Q=0$ ). Unlike the case of the Shkadov's integral approach, only disturbances with  $Q=1/2$  generate new types of the steady-state traveling solutions in the case of the Navier-Stokes equations.

## REFERENCES

- [1] B. Scheid, C. Ruyer-Quil and P. Manneville, "Wave Patterns in Film Flows: Modeling and Three-Dimensional Waves", *J. Fluid Mech.*, vol. 562, pp. 183–22, (2006).