Secondary bifurcation of mixed-cross-spirals connecting different travelling wave solutions
- BIFD2011 -

S. Altmeyer*, Ch. Hoffmann*

* Institut für Theoretische Physik,
Universität des Saarlandes, D-66123 Saarbrücken, Germany
e-mail: sepp@lusi.uni-sb.de

ABSTRACT

The fluid between two independently rotating, concentric cylinders exhibits numerous transitions between different flow states. The simplest of them are standing and travelling waves, both bifurcating forward out of the basic state. Due to the geometry, travelling waves are mostly realized by so-called spirals.

We present numerical results of secondarily forward bifurcating, stationary flow states that mediate transitions between these travelling helical waves with different azimuthal wave numbers. These so-called mixed-cross-spirals (MCS) can be seen as nonlinear superpositions of the involved pure spiral solutions as indicated in the figure. Thereby, the contribution of the respective spiral component to the entire MCS varies continuously with the control parameters. In that notation, the well-studied cross-spirals represent a special case of MCS as they consist of two *mirror-symmetric* spiral components.

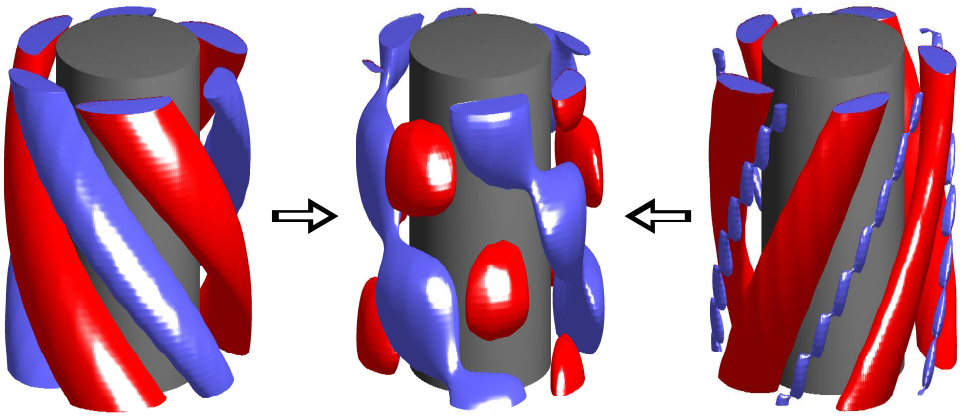

Figure 1: Iso-surfaces of azimuthal vorticity of a numerically calculated MCS (middle) which mediates the transition between two spirals of different azimuthal wave numbers M, i.e. the $M = 3$ left-winding spiral (left) and the $M = 5$ right-winding spiral (right).

Two bifurcation scenarios have been found: MCS bifurcate as a stable (unstable) solution out of a stable (unstable) spiral branch. In the first case, the stable SPI then becomes unstable. This behaviour is completely unaffected by the stability behaviour of other solutions.

Generally, the occurrence of MCS seems to be a consequence of the periodic boundary conditions which, in contrast to rigid axial lids or other geometries, allow for multi-stable spiral solutions with different azimuthal wave numbers and also for multi-stability of different MCS. Due to wave number selection effects in longer axial periodic systems, multi-stability regions seem to disappear mostly and it is an open question whether such regions then exist at all.

REFERENCES

[1] S. Altmeyer, Ch. Hoffmann *Secondary bifurcation of mixed-cross-spirals connecting travelling wave solutions*, New J. Phys. **12**, 113035, 2010.

[2] H. L. Swinney, J. P. Gollubitsky *Hydrodynamic Instabilities and the Transition to Turbulence*, (Berlin: Springer), 1981.

[3] M. Golubitsky, I. Stewart D. Schaeffer *Singularities and Groups in Bifurcation Theory II*, (New York: Springer), 485512, 1988.

[4] R. Tagg, *The Couette-Taylor Problem*, Nonlinear Science Today, **4(3)**, 1-25, 1994.