Optimal $\mathcal{H}_2$ Norm Model Order Reduction for LPV Systems by Global Galerkin Projection

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ABSTRACT

In this paper, we focus on the optimal $\mathcal{H}_2$ model order reduction for linear parameter-varying (LPV) systems by a global Galerkin projection method. Consider the LPV system

$$\Sigma(\mathbf{p}) : \begin{cases} \dot{x} = A(\mathbf{p}(t))x + B(\mathbf{p}(t))u \\ y = C(\mathbf{p}(t))x + D(\mathbf{p}(t))u, \end{cases}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^q$ is the output and $\mathbf{p}(\mathbf{t}) : \mathbb{R} \mapsto \mathbb{R}^{n_p}$ is the time-varying parameter. When the parameter trajectory is known as $\mathbf{p}(t) = \rho(t), t \in [t_0, t_f]$, We give a formal definition of the $\mathcal{H}_2$ system norm in the frequency domain and a computational scheme in the time domain. Based on this computational scheme, the global $\mathcal{H}_2$ model order reduction problem can be formulated as a Riemannian optimization problem:

$$\begin{aligned} &\min_{V} \frac{1}{2}\mathrm{trace}\left(\int_{\mathrm{t_0}}^{\mathrm{t_f}} \mathrm{C}(\rho(\mathrm{t}))\left(\mathcal{R}_{\mathrm{f}}(\mathrm{t}) + \mathrm{V}\mathcal{R}_{\mathrm{r}}(\mathrm{t})\mathrm{V}^\top(\mathrm{t}) - 2\mathrm{X}(\mathrm{t})\mathrm{V}^\top\right)\mathrm{C}^\top(\rho(\mathrm{t}))\mathrm{dt}\right) \\ &\text{subject to } V^\top V = I_r, \end{aligned}$$

where V is the projection matrix that needs to be found. Let $\Sigma_r(\mathbf{p})$ denote the reduced-order model, the matrix

$$\begin{pmatrix} \mathcal{R}_{\mathrm{f}}(t) & X(t) \\ X^\top(t) & \mathcal{R}_{\mathrm{r}}(t) \end{pmatrix}$$

is called the time-varying reachability Gramian of the error system $\Sigma(\mathbf{p}) - \Sigma_r(\mathbf{p})$.

A Riemannian conjugate gradient method is proposed to solve the optimization problem. A small-scale numerical example is tested to demonstrate the performance.

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