Numerical approximation of Bayesian Inverse Problems for PDEs by Reduced-Order Modeling techniques

Andrea Manzoni* **and Stefano Pagani†**

* Ecole Polytechnique Fédérale de Lausanne
Mathematics Institute – Chair of Modeling Scientific Computing
Station 8, CH-1015 Lausanne, Switzerland
e-mail: andrea.manzoni@epfl.ch

† Ecole Polytechnique Fédérale de Lausanne
Mathematics Institute – Chair of Modeling Scientific Computing
Station 8, CH-1015 Lausanne, Switzerland
e-mail: stefano.pagani@epfl.ch

ABSTRACT

The solution of inverse problems involving systems modeled by partial differential equations (PDEs) is computationally demanding. We show how to take advantage of reduced-order modeling (ROM) techniques, such as the reduced basis method for parametrized PDEs [1, 2], to speed up the numerical approximation of Bayesian inverse problems related with parameter estimation for both stationary and time-dependent PDEs. In the former case, we rely on Markov Chain Monte Carlo (MCMC) methods to characterize the posterior distribution of the parameters [3]; in the latter, we exploit the Ensemble Kalman filter for performing state/parameter estimation sequentially [4]. In both cases, we replace usual high-fidelity techniques (such as the finite element method) with inexpensive but accurate reduced-order models to speed up the solution of the forward problem. On the other hand, we develop suitable ROM error surrogates (ROMES) to quantify in an inexpensive way the error between the high-fidelity and the reduced-order approximation of the forward problem, in order to gauge the effect of this error on the posterior distribution of the identifiable parameters [5, 3, 4]. Numerical results dealing with the estimation of both scalar parameters and parametric fields highlight the combined role played by RB accuracy and REM effectivity [6].

REFERENCES

[1] A. Quarteroni, A. Manzoni, and F. Negri. *Reduced Basis Methods for Partial Differential Equations. An Introduction*, volume 92 of Unitext Series. Springer, 2016.

[2] F. Negri, A. Manzoni, and D. Amsallem. Efficient model reduction of parametrized systems by matrix discrete empirical interpolation method. J. Comput. Phys. **303**:431–454, 2015.

[3] A. Manzoni, S. Pagani, T. Lassila. Accurate solution of Bayesian inverse uncertainty quantification problems combining reduced basis methods and reduction error models. SIAM/ASA J. Uncertainty Quantification **4**(1): 380–412, 2016

[4] S. Pagani, A. Manzoni, A. Quarteroni. A reduced basis ensemble Kalman filter for state/parameter identification in large-scale nonlinear dynamical systems, 2016. Mathicse Report 18.2016.

[5] M. Drohmann, K. Carlberg. The ROMES Method for Statistical Modeling of Reduced-Order-Model Error. SIAM/ASA J. Uncertainty Quantification **3**(1): 116–145, 2015.

[6] S. Pagani. Reduced-order models for inverse problems and uncertainty quantification in cardiac electrophysiology. PhD Thesis, Politecnico di Milano, 2017.