A new equilibrated residual method: improving accuracy and efficiency of flux-free error estimates in two and three dimensions

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ABSTRACT

A new methodology to compute guaranteed upper bounds for the energy norm of the error in the context of linear finite element approximations of the reaction-diffusion equation introduced in [2] is presented. The new approach revisits the ideas in [1] with the goal of substantially reducing the computational cost of the *flux-free* (FF) method while retaining the good quality of the bounds. The new methodology provides also a technique to compute equilibrated boundary tractions improving the quality of standard equilibration strategies. The *zeroth-order* equilibration conditions are imposed using an alternative less restrictive form of the *first-order equilibration* conditions, along with a new efficient minimization criterion. This new equilibration strategy provides much more accurate upper bounds for the energy and requires only doubling the dimension of the local linear systems of equations to be solved.

The following figure shows the results obtained for the 2D Poisson equation in the standard L-shaped domain with a reentrant corner. The effectivity indices are shown both for existing flux-free and equilibrated standard error estimates and for the new presented techniques.



Finally, the presented equilibrated residual method is extended to the three-dimensional setting.

REFERENCES

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