Uncertainty Modeling of Spatially Invariant Distributed Dynamical Systems

Marko Jokic*, **Bruno Dogancic**†, **Andrej Jokic**† **and Mihael Lobrovic**†

* † Faculty of Mechanical Engineering and Naval Architecture
University of Zagreb
Ivana Lucica 5, 10000 Zagreb, Croatia
e-mail: {mjokic, bdogancic, ajokic, mlobrovic}@fsb.hr, web page: http://www.fsb.hr/

ABSTRACT

Spatially distributed dynamical systems, i.e. dynamical systems comprised of a number of interconnected (coupled) and mutually interacting individual dynamical subsystems, have gained significant importance in science and engineering throughout the last decades – examples include smart structures, smart electrical power systems, multi-agent robotic systems, networks of embedded systems and so on. Modeling, analysis and control of spatially distributed systems has been recognized as a challenging task, and has attracted significant research interest [1].

This paper is focused on a particular class of spatially distributed systems, namely spatially invariant distributed systems in the sense that the overall system is comprised of identical subsystems interconnected in such way that a translation in spatial direction by one or more subsystem instances does not change the overall system dynamics. Typical examples include discretized infinite-dimensional structural systems such as beams and plates, structures assembled of identical elements and so on. To be more specific, the presented research tries to answer the following: When is a spatial discretization of single subsystem "good enough" such that the overall system dynamics is accurately described [2]? Furthermore, how do we model individual subsystem uncertainty such that it contains information relevant for the overall system?

To illustrate the results, we present an example of spatially invariant distributed dynamical system comprised of a series of simply-supported Euler beams coupled with (possibly uncertain) springs and viscous dampers. For each beam, a series of successively finer spatial discretized models is constructed, as well as an uncertainty model describing a discretization error. We stress that this series of successively "better" models of a single beam comprised of discretized models and corresponding uncertainties are essentially local, in the sense that they do not take into the account that the overall system is comprised of a series of coupled individual instances. Furthermore, to evaluate which level of discretization is sufficient to accurately describe overall system dynamics, we construct a reduced order model which, in the worst case sense, approximates the overall system. Then, a series of successively better beam models are evaluated using such reduced order model. When doing so, frequency weights of individual uncertainty models are adjusted such that the uncertainty information that does not affect the overall system dynamics is discarded.

Acknowledgment: This work has been fully supported by Croatian Science Foundation under the project number 9354, Control of Spatially Distributed Systems.

REFERENCES

[1] Jovanovic, M.R. *Modeling, analysis, and control of spatially distributed systems*. PhD thesis, University of California, Santa Barbara, 2004.

[2] Jones, B.L. and Kerrigan, E.C. When is the discretization of a spatially distributed system good enough for control?. *Automatica*, Vol. **46**(9), pp. 1462–1468, 2010.