Crack Propagation in Brittle Materials via Generalized Ambrosio-Tortorelli Models

Nicola Ferro[†], **Stefano Micheletti**[*,†], **Simona Perotto**[†] **and Marianna Signorini**[†]

[*,†] MOX - Dipartimento di Matematica, Politecnico di Milano
Piazza L. da Vinci, 32, I-20133 Milano, Italy
e-mail: {nicola.ferro, stefano.micheletti, simona.perotto, marianna.signorini}@polimi.it

ABSTRACT

The minimization of the Ambrosio-Tortorelli (AT) functional is often employed to study crack propagation in brittle materials. The crack is identified by a smooth phase-field, and the minimization of the functional by numerical methods is obtained by solving elliptic boundary value problems, via a finite element method. Typically, the crack thickness is very small compared to the domain size, so that the computational cost is very high, due to the need of thoroughly capturing features on the crack scale. An efficient approach to tackle this issue can be based on mesh adaptation, driven by an a posteriori error estimator, which allows one to sharply refine the grid only in a thin neighborhood of the crack (see, e.g., [5]). In [1, 2, 3], an anisotropic error estimator and a new minimization algorithm are proposed and applied to the classical AT approximation, in the case of both anti-plane and plane-strain isotropic linear elasticity. Several numerical tests assess the reliability of the whole adaptation procedure. Actually, the employment of an anisotropic grid allows one to further reduce the number of mesh elements in comparison with isotropic adaptation techniques. In this communication, we extend the approach presented in [1, 2, 3] to the generalized AT model considered in [5], where the AT functional is modified in order to deal with more general constitutive laws. Moreover, we address the extension proposed in [4] for including thermal inelastic effects, which model both genesis and propagation of cracks in brittle materials. Good accuracy as well as significant computational saving guaranteed by the resulting adaptive algorithm are confirmed by verification on standard benchmarks and on a (simplified) disc brake geometry.

REFERENCES

[1] Artina, M., Fornasier, M., Micheletti, S. and Perotto, S. Anisotropic mesh adaptation for crack detection in brittle materials. *SIAM J. Sci. Comput.*, Vol. **37(4)**, pp. B633–B659, (2015).

[2] Artina, M., Fornasier, M., Micheletti, S. and Perotto, S. Anisotropic adaptive meshes for brittle fractures: parameter sensitivity. Numerical Mathematics and Advanced Applications. Series: Lect. Notes Comput. Sci. Eng., Vol. **103**, Springer, pp. 293–302, (2015).

[3] Artina, M., Fornasier, M., Micheletti, S. and Perotto, S. The benefits of anisotropic mesh adaptation for brittle fractures under plane-strain conditions. New challenges in grid generation and adaptivity for scientific computing, SEMA SIMAI Springer Series, Vol. **5**, Springer, pp. 43–67, (2015).

[4] Bourdin, B., Marigo, J.-J., Maurini, C. and Sicsic, P. Morphogenesis and propagation of complex cracks induced by thermal shocks. *Phys. Rev. Lett.*, Vol. **112(1)**, pp. 014301-1–014301-5, (2014).

[5] Burke, S., Ortner, C. and Süli, E. An adaptive finite element approximation of a generalized Ambrosio-Tortorelli functional. *Math. Models Methods Appl. Sci.*, Vol. **23(9)**, pp. 1663–1697, (2013).