## A Multi-rate Numerical Method for Systems of Partial Differential Equations

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## ABSTRACT

Modeling complex, multi-physics problems often involves dealing with a large range of time and space scales. In order to efficiently solve the resulting system of PDEs, it is essential to take advantage of these different scales.

We present a multi-rate numerical method originally devised to solve systems of ODEs [1] and recently adapted to solve systems of conservation laws [2]. This method uses a classic ODE error estimator for each component of the solution vector in order to locally adapt the time step. We discuss basic properties of the method and focus on the choice of error estimators. We then present a new, hybrid method by coupling the multi-rate method with a posteriori error estimators for PDEs [3].

We present numerical results on a two-phase flow in porous media problem and discuss future improvements of the method and its possible coupling with Adaptive Mesh Refinement (AMR) methods.

## REFERENCES

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