PGD for Constrained Parametric Space with a Non-intrusive Implementation

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ABSTRACT

Proper generalized decomposition (PGD) is a powerful technique on tackling the so-called *curse of dimensionality* by means of reduced order modelling. Based on the idea of separation of variables, the parametric solution is assumed to have a separated representation, in order to reduce the multi-dimensional problem to a sum of tensor product of functions defined in lower dimensional subspaces. In a practical implementation, PGD generates a computational vademecum in the offline phase, which can be frequently used in the subsequent online phase for different types of problems that require multi-queries and/or extremely fast responses.

In this work, focused on the offline phase, we propose a novel strategy on the separation for the parametric problems. In many cases, the parametric space is not Cartesian, thus has low separability. For example, when taking the loading location as extra-coordinates, the admissible locations may usually be a non-uniformly curved surface. Instead of separating the parametric space as a tensor product of 1D subspaces which may lead to non-physical solutions, we separate it into a tensor product of higher dimensional (2D or 3D) parametric subspaces which collect the highly correlated parameters. With this collective strategy of keeping some correlated parameters unseparated, we are able to solve problems involving parametric spaces that have low separability due to constraints from physics, geometry or other aspects. Besides, the convergence rate is improved due to the higher dimension of the separated parametric subspaces.

Most PGD frameworks currently available in literatures are based on intrusive implementations with academical codes, while PGD algorithms are not usually implemented in commercial software commonly used in industry. Motivated by accelerating the coding procedure of PGD and extending its industrial applications, we propose a non-intrusive PGD scheme. It takes advantage of commercial solver as a black box, and solves the sectorized problems outside of in-house developed PGD codes.

In this work, we present a practical application in bone mechanics based on linear elasticity. We used PGD vademecum for parameter identification of biological tissues from experimental data, and for patient-specific real-time simulation in clinical decision making.

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