The Reduced Basis-Hierarchical Model Reduction Approach for nonlinear PDEs and kinetic transport equations

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ABSTRACT

The main focus of this talk is the extension of the Reduced Basis-Hierarchical Model Reduction approach (RB-HMR) to nonlinear Partial Differential Equations (PDEs) and kinetic transport equations. The key idea of HMR is to perform a Galerkin projection onto a reduced space, which combines the full solution space in a dominant direction with the span of modal orthonormal basis functions in the transverse direction. While so far the modal basis functions in the HMR approach have been chosen a priori, e.g. as polynomials [3], we employ a highly nonlinear approximation for the construction of the basis [1, 2, 4]. To this end, we first derive a parametrized problem in the transverse direction from the full problem where the parameters reflect the influence from the unknown solution in the dominant direction. The HMR basis is then selected via RB methods from snapshots of this lower-dimensional parametrized PDE.

In order to deal with nonlinear PDEs we propose to approximate also the nonlinear operator with an expansion in an orthonormal basis on the transverse direction, which is also constructed in a highly nonlinear fashion [4]. Run-time experiments verify a linear scaling of the method in the number of degrees of freedom used for the computations in the dominant direction. Finally, we reduce the dimension of kinetic transport equations by a truncated basis expansion in the velocity variable, obtaining a hyperbolic system of equations in space and time. Unlike former methods like the Legendre moment models, the new framework generates a suitable problem-dependent basis of the reduced velocity space that mimics the shape of the solution in the velocity variable [1].

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