Efficiency analysis of patch size and type for Error Estimates based on implicit residual and local Dirichlet patch problems.

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ABSTRACT

The work presented starts from 3 known Finite Element Method error estimates, based on reducing the Residual to local Dirichlet Problems. The original estimates are 'partition of unity using hat functions', 'space enhancement' and 'subspace projection' which can be found in I. Babuska and A. Miller [1], K.Runesson and co-workers [2] and A.Huerta and co-workers [3] respectively. One of the present work's aims is to fit these methods in a common framework that allows compare its performance. Then it is also possible to combine concepts already used in the tested methods, in an attempt to improve them.

The general residual's expression is $a(e, v) = l(v) - a(u^h, v) =: R(v)$, being $a(\cdot,\cdot)$ and $l(\cdot)$ the usual bilinear form and bounded linear functional derived from a weak formulation of an elliptic boundary value problem on a Hilbert space. Using an analogy of the residual as a distributed load and splitting its integral on elemental contributions for each patch ($R(v) = \sum_{k=1}^{ne} R_k$) leads to a set of global problems, with no approximation but extra cost equal to the number of contributions. Our work checks if the gradient of this set of global fields (which is the value inside the bilinear form) vanishes far from the locally load-like applied residual. This verification allows an approximation consisting in the reduction of these global problems neglecting the part where the contribution is of a much lesser order. This local patches can be arbitrary shaped as long as the vanishing assumption is satisfied. The technique also avoids the necessity of defining non obvious average rules, since each local contribution is a part of an addition to the total estimation of the error.

This new approach is then used on the methods described on [1] and [2]. The performance of all the estimates using h and p refinement, and different patches when possible is compared in terms of accuracy and computational cost using the same test problem.

The idea of 'subspace projection', is also used to define a two steps method. The first step computes the elemental boundary with p refinement and a large patch. Once this values are computed, they are applied as Dirichlet B.C. for a bubble problem taking advantage of the orthogonality shown in the original method described in [3].

The variations introduced showed similar performance for the first method and better performance for the second one. What may be more relevant though, is the flexibility in terms of patch definition and type of enhancement. The 2 steps method produced better accuracy than the interpolation of the 1st step solution, but the extra cost may not be worthy.

REFERENCES

[1] I. Babuska and A. Miller. The post-processing approach in the finite element method. Part 1. *Internat. J. numer. Methods Engrg.*, 20;1085-1109, 1984.

[2] F.Larsson, P.Hansbo and K. Runesson, Strategies for computing goal-oriented a posteriori error measures in non linear elasticity *Internat. J. numer. Methods Engrg.*, 55;879-894, 2001.

[3] P. Diez, L.Egozcue and A. Huerta. A posteriori error estimation for standard finite element analysis *Comput. Methods Appl. Engrg.*, 163,141-157 1998.