Goal-oriented adaptivity for the Generalized Multiscale Finite Element Method

Sara Pollock*, Eric T. Chung†, Wing Tat Leung†† and Sai-Mang Pun†

* Department of Mathematics & Statistics
Wright State University
Dayton, OH, USA
e-mail: sara.pollock@wright.edu, web page: http://www.wright.edu/∼sara.pollock

† Department of Mathematics
The Chinese University of Hong Kong
Shatin, Hong Kong

†† Department of Mathematics
Texas A&M University
College Station, TX, USA

ABSTRACT

Goal-oriented adaptivity is introduced within the framework of the Generalized Multiscale Finite Element Method (GMsFEM) for problems of high-contrast flow. GMsFEM is a flexible framework for approximating solutions to problems with multiple scales and high contrast. The methodology is based on two mesh partitions: a coarse mesh over which the global problem is considered, and a subordinate fine mesh over which only local problems are considered. First, multiscale basis functions are computed through the solution of local fine-scale problems to capture the fine-scale heterogeneities. Then, an ordered selection of these basis functions in each coarse neighborhood is used to approximate the global solution.

The role of *a posteriori* error estimation and adaptivity in GMsFEM is to identify which coarse neighborhoods to enrich with additional multiscale basis functions, in the interest of both accuracy and efficiency. Goal-oriented error indicators are introduced to increase the performance of the simulation when seeking a given quantity of interest, which is generally a localized function of the weak solution. Our results show that in both standard and mixed formulations, goal-oriented error estimation outperforms standard adaptivity in the approximation of the quantity of interest.

Here we will discuss the formulation of goal-oriented *a posteriori* error indicators within GMsFEM. We will consider how both residual-based and dual-weighted residual methods can be extended to this framework. The techniques will be illustrated with numerical examples.

REFERENCES

[1] Efendiev, Y., Galvis, J. and Hou, T. Y. Generalized multiscale finite element methods (GMsFEM). *Journal of Computational Physics* Vol. **251**, pp. 116–135, (2013).

[2] Chung, E. T., Leung, W. T. and Pollock, S. Goal-oriented adaptivity for GMsFEM. *Journal of Computational and Applied Mathematics* Vol. **296**, pp. 625–637, (2016).

[3] Chung, E. T., Pollock, S. and Pun, S. M. Goal-oriented adaptivity of mixed GMsFEM for flows in heterogeneous media. *Submitted*, (2016).