Space-time variational formulations and a posteriori error estimation of transport equations

Julia Brunken*, **Kathrin Smetana***, **and Karsten Urban**†

* Institute for Computational and Applied Mathematics
University of Münster
Einsteinstr. 62, 48149 Münster, Germany
e-mail: julia.brunken@uni-muenster.de, kathrin.smetana@uni-muenster.de

† University of Ulm
Institute for Numerical Mathematics
Helmholtzstr. 20, 89081 Ulm, Germany
e-mail: karsten.urban@uni-ulm.de

ABSTRACT

In this talk we present a posteriori error bounds based on stable space-time variational formulations for transport equations.

In the context of model reduction for time-dependent transport equations, efficient error estimators are desirable both for the construction of reduced models and the validation of reduced solutions. To derive such estimates it is beneficial to consider space-time variational formulations: these may lead to favorable inf-sup constants, which is important as their inverse enters into error estimates as considered in the Reduced Basis context. Starting from stable variational formulations for stationary first order transport equations as developed in [1] we consider the corresponding time-dependent problem. Here, we prove well-posedness of the space-time variational formulation by using ideas from [2, 3]. By doing so, we obtain a computable lower bound for the space-time inf-sup constant which then can be used for error estimators. Note that while in the parabolic case as considered in [2, 3] the spatial operator corresponds to a bilinear form that satisfies a Gårding inequality and is defined on coinciding ansatz and test spaces, here we consider an inf-sup stable spatial problem with different ansatz and test spaces. Numerical experiments confirm the theoretical findings.

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