Adaptive optimal control of contact problems

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ABSTRACT

In this talk, we present a-posteriori error estimations in context of optimal control of contact problems; in particular of Signorini's problem. Due to the contact side-condition, the solution operator of the underlying variational inequality is not differentiable, yet we want to apply Newton's method. Therefore, the non-smooth problem is regularized by penalization and afterwards discretized by finite elements.

We derive optimality systems for the regularized formulation in the continuous as well as in the discrete case. This is done explicitly for Signorini's contact problem, which covers linear elasticity and linearized surface contact conditions. The latter creates the need for treating trace-operations carefully, especially in contrast to obstacle contact conditions, which exert in the domain.

These systems are used to define an MPEC-Lagrangian that sets the goal functional of the optimal control problem in relation to the optimality systems. We achieve representations for the regularization, discretization and numerical error by employing the dual weighted residual (DWR) method onto the objective, which is measured by the goal functional of the optimal control problem.

In a next step, we develop error estimators for those errors. For this purpose, we differentiate the solution operator of the discrete, regularized problem with respect to the penalization parameter. Assuming that the matrix in Newton's method is regular, we prove that we obtain a higher order approximation regarding the penalization. The evaluation of the derivate is done by an additional Newton step to a different right hand side.

The resulting error estimator for regularization error is evaluated only in the contact area. Therefore its computational cost is especially low for Signorini's contact problem. By applying standard reconstruction techniques for patches, we gain a suitable approximation concerning the discretization, too. The numerical error estimator is the usual by-product of the standard DWR-techniques.

Finally, we utilize the estimators in an adaptive refinement strategy balancing regularization and discretization errors. Numerical results substantiate the theoretical findings.