Unconventional Error Representations for Goal Oriented p-Adaptivity and its Applications

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ABSTRACT

An increasingly number of engineering applications require the so-called goal-oriented adaptivity. For instance, geophysical problems often need to approximate a specific quantity of interest. Goal-oriented algorithms optimise the error of a given quantity of interest by minimising an upper bound of such error by performing local refinement. The upper bound of the error in the quantity of interest is expressed in terms of an element-wise representation over the entire computational domain.

To represent the error in a quantity of interest, most authors derive local and computable quantities via the global bilinear form associated to the adjoint problem. However, our methodology introduces an alternative bilinear form such that its intrinsic properties are more convenient than the original ones (e.g. positive definiteness). A new representation of the error can be obtained using the Riesz representation of the residual error functional of the adjoint problem via this alternative form. Then, new upper bounds of the error on the quantity of interest can be computed similarly than with the classical approach.

In this presentation, we show that a properly selected alternative form improves of the sharpness of the upper bounds of the error representation in the pre-asymptotical regime. Moreover, the proposed method generalises the classical method, which corresponds to the case when the alternative bilinear form is selected as that given by the adjoint problem. Due to the general formulation of the method, it can be applied to a wide range of 1D, 2D and 3D problems.

We describe numerically the main features and limitations of the proposed method with multi-dimensional acoustic examples and applications. Extensive numerical results are illustrated using uniform h- and p-refinements, as well as a simple self-adaptive goal-oriented p-refinement strategy. The application to other adaptive algorithms such as a goal-oriented hp-adaptive algorithm, adaptivity in a high continuity space, or adaptivity in time domain is straightforward.

REFERENCES

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