An adaptive algorithm for the time dependent transport equation with anisotropic finite elements and the Crank-Nicolson scheme

Samuel Dubuis*, Marco Picasso†

* Mathematics Institute of Computational Science and Engineering (MATHICSE)
Ecole Polytechnique Fédérale de Lausanne
Station 8, 1015 Lausanne, Switzerland
e-mail: samuel.dubuis@epfl.ch, web page: http://asn.epfl.ch

† Mathematics Institute of Computational Science and Engineering (MATHICSE)
Ecole Polytechnique Fédérale de Lausanne
Station 8, 1015 Lausanne, Switzerland
e-mail: marco.picasso@epfl.ch, web page: http://asn.epfl.ch

ABSTRACT

The time dependent transport equation is solved with stabilized continuous, piecewise linear finite elements and the Crank-Nicolson scheme [1]. Finite elements with large aspect ratio are advocated in order to account for boundary layers. The error due to space discretization has already been studied in [2]. Here, the error due to the use of the Crank-Nicolson scheme is taken into account. Anisotropic a priori and a posteriori error estimates are proved. The a posteriori upper bound is obtained using a quadratic reconstruction in time as in [3].

The quality of the error estimator is first validated on non adapted meshes and constant time steps. An adaptive algorithm in space and time is then proposed, with goal to build a sequence of anisotropic meshes and time steps, so that the final error is close to a preset tolerance. Numerical results on adapted, anisotropic meshes and time steps show the efficiency of the method.

REFERENCES

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