Goal Oriented Time Adaptivity Using Local Error Estimates

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ABSTRACT

When solving ODEs or PDEs, one is not always interested in the solution $\mathbf{u}(t)$, but rather a functional of the solution. Starting from an Initial Value Problem (semidiscretized PDE) with solution $\mathbf{u}(t)$, we consider functionals of the form

$$J(\mathbf{u}) = \int_0^T j(\mathbf{u}(t))\, dt$$

with $j : \mathbb{R}^n \to \mathbb{R}$. Examples for this are determining the net-flow of mass in an engine, the average energy production of a turbine or the drag coefficient for a vehicle.

The standard approach for controlling the error in the quantity of interest $J(\mathbf{u})$ is the dual-weigthed residual method [1]. To obtain an estimate the error in the quantity of interest, this method requires solving the given ODE (PDE) forward in time and its adjoint problem backwards in time, multiple times each, to reach a desired precision.

An alternative approach is to use time-adaptive schemes based on local error estimates [2], which require only one forward solve, at the cost of losing the error estimate in $J(\mathbf{u})$.

We propose a new method to get the best of both worlds, where we aim to reach a desired precision in a single forward solve. To this end, we take the local error approach, but determine the timesteps using only the quantities that are relevant for $J(\mathbf{u})$.

In the talk, we will present an analysis of this method and numerical results of the new method, which compare it to the dual-weighted residual method and classical time-adaptivity based on local error estimates.

REFERENCES

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