Recovery of equilibrium from compatible finite element models of plates

Edward A.W. Maunder*, J.P. Moitinho de Almeida†

* College of Engineering, Mathematics and Physical Sciences
University of Exeter, North Park Road, Exeter, EX4 4QF, UK
Email: e.a.w.maunder@exeter.ac.uk, web page: http://www.exeter.ac.uk

† Department of Civil Engineering, Architecture & Georesources
IST, University of Lisbon, Portugal
Email: moitinho@civil.ist.utl.pt, web page: http://www.tecnico.ulisboa.pt

ABSTRACT

It has long been recognized that dual pairs of finite element solutions provide a sound basis for error estimation and adaptivity, and new alternative partition of unity (PU) techniques for deriving one form of solution from the other have been proposed [1], without the need for complete analyses of both compatible and equilibrating models.

In this presentation we extend these techniques for recovering strong forms of equilibrium from compatible finite element models of 2D and 3D continua to similar models of plate bending problems. As with the technique for continua, we consider only simplicial elements where the partition of unity functions are piecewise linear. Plate behaviour is assumed to be governed by either Reissner-Mindlin' first order shear deformation theory, or Kirchhoff's classical plate theory for thin plates where shear deformation is neglected.

We start with compatible Reissner-Mindlin elements based on the standard isoparametric formulation of polynomial degree d_θ for rotations and degree d_w for transverse deflections. The PU technique relies on the formation of fictitious loads on a star patch of elements which are derived from the moment and shear fields of a conforming element. For elements of degree $d_\theta > 1$, these loads together with the partitioned prescribed loads on the patch are perfectly balanced and present a well-posed problem for local analysis using hybrid equilibrium elements [2]. The equilibrated patch solutions are then superimposed to achieve the total equilibrated solution.

However, for elements of degree $d_\theta = 1$, the fictitious loads so derived only lead to transverse equilibrium of a star patch, and corrections are required to regain rotational equilibrium as well. The corrective process does require extra local analyses of a more general form of patch, as was the case with models of continua, and this process, which involves the use of appropriate mixed forms of element, is also presented.

The Reissner-Mindlin elements are more suitable for modelling plates with medium thickness since they are susceptible to locking for very thin plates, particularly for low degrees. It is then appropriate to next consider conforming models based on Kirchhoff's theory. However, for complete compatibility we need to turn to elements of at least degree $d_w = 5$ [3]. The fictitious loads then lead to overall equilibrium of a star patch. For complete equilibrium of its stress fields, as determined by analyses with hybrid equilibrium models, the degree of the hybrid must be at least 4.

REFERENCES

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