POD with collocation on the fly

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ABSTRACT

An adaptive method to accelerate time-dependent solvers for PDEs has been proposed [1,2,3] that adaptively combines on the fly a standard numerical solver and a low-dimensional system. The latter, in turn, combines POD (to identify modes) and Galerkin projection.

To improve the computational efficiency of the method, it is essential to use a reduced number of points (instead of the whole computational mesh used in the spatial discretization) to both carry out POD and to perform the Galerkin projection of the equations.

In this work, we propose a novel collocation method for the POD of a set of snapshots. The main idea is to apply the classical LU decomposition with pivoting to the snapshots matrix to select both a set of collocation points and a set of most appropriate snapshots. Then, POD is performed on the selected snapshots using an inner product based on the collocation points. This synergic combination of LU decomposition and POD (referred to as the LUPOD method [4]) provides a computationally inexpensive method whose accuracy is comparable to that of standard POD.

The performance of the combination of LUPOD with Galerkin projection will be tested on the complex Ginzburg-Landau equation. This is a simple nonlinear PDE depending on three parameters that exhibits quite complex dynamics, resulting from the many nontrivial instabilities and bifurcations that occur as the parameter values are varied. We will show how the above mentioned ideas can be exploited to construct bifurcation diagrams in the spirit of [5].

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