A Consistent Bayesian Approach for Stochastic Inverse Problems

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ABSTRACT

Uncertainty is ubiquitous in computational science and engineering. Often, parameters of interest cannot be measured directly and must be inferred from observable data. The mapping between these parameters and the measureable data is often referred to as the forward model and the goal is to use the forward model to gain knowledge about the parameters given the observations on the data. Statistical Bayesian inference (see e.g., [1, 2]) is the most common approach for incorporating stochastic data into probabilistic descriptions of the input parameters. This approach uses data and an assumed error model to inform posterior distributions of model inputs and model discrepancies. An explicit characterization of the posterior distribution is not necessary since certain sampling methods, such as Markov Chain Monte Carlo, can be used to draw samples from the posterior.

We have recently developed an alternative Bayesian solution to the stochastic inverse problem based on the measure-theoretic principles developed in [3]. We prove that this approach, which we call consistent Bayesian inference, produces a posterior distribution that is consistent in the sense that the push-forward probability density of the posterior through the model will match the distribution on the observable data, i.e., the posterior is consistent with the model and the data [4]. Our approach only requires approximating the push-forward probability density of the prior through the computational model, which is fundamentally a forward propagation of uncertainty. Numerical results will be presented to highlight various aspects of the consistent Bayesian approach, to compare with the statistical Bayesian approach, and to demonstrate the effectiveness of adaptive sampling strategies.

REFERENCES

[1] M. Kennedy and A. O'Hagan, "Bayesian calibration of computer models", Journal of the Royal Statistical Society: Series B (Statistical Methodology), Vol. 63, pp. 425-464, (2001).

[2] A. M. Stuart, "Inverse problems: A Bayesian perspective", Acta Numerica, Vol. 19, pp. 451–559, (2010).

[3] J. Breidt, T. Butler, and D. Estep, "A computational measure theoretic approach to inverse sensitivity problems I: Basic method and analysis", SIAM J. Numer. Analysis, 49, pp. 1836–1859, (2012).

[4] T. Butler, J. Jakeman and T. Wildey, "A consistent Bayesian formulation for stochastic inverse problems based on push-forward measures", Submitted to SIAM J. Sci. Comput. (2016).