A Higher-Order Fictitious Domain Method using the Marching Volume Polytopes Algorithm

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ABSTRACT

We present a variant of the fictitious domain method, where the characteristic function for the domain of interest is defined via elements resulting from the Marching Volume Polytopes Algorithm proposed in [3]. The resulting scheme is tested with a model problem and applied to the simulation of a NC-milling process and of gravity acting on a given section of a mountain range.

As usual in fictitious domain methods, the actual domain for the problem under consideration is embedded in some covering domain that is represented by some simple, e.g. Cartesian, grid. The latter or some octree-like refinement of the latter yields a FEM-grid to be employed for the definition of higher-order tensor product finite element shape functions based upon Lagrange or integrated Legendre polynomials. Possible (multi-level) hanging nodes and varying polynomial degrees in the chosen FEM-grid are handled with so called connectivity matrices to ensure a H^1-conforming discretization, see [1]. To include the actual domain of interest into the discretization, the shape functions are multiplied with a characteristic function for that domain. We have chosen a characteristic function with range $\{0, 1\}$ instead of $\{\varepsilon, 1\}$ for some $\varepsilon > 0$. To improve the conditioning of the resulting systems of equations, we omit degrees of freedom with too small support, see [2]. The actual definition of the characteristic function is done via elements resulting from an additional octree-like refinement of the FEM-grid towards the surface of the domain of interest, followed by (possibly multiple) applications of the Marching Volume Polytopes Algorithm.

The presentation is concluded with a numerical verification of the proposed scheme, along with two challenging applications that greatly benefit from the decoupling of the domain resolution from the actual finite element discretization via the characteristic function. In the roughing process of NC milling, a significant amount of heat is induced into a work piece due to the conversion of energy during the chip formation process. This results in global thermo-mechanical deformations that remain present in the subsequent finishing process and may consequently cause a strong deviation of the work piece surface from its designed shape after cooling down. Thus, critical manufacturing tolerances may be exceeded. We will show that the proposed scheme is able to predict the characteristics and magnitude of these deviations, see [2]. For the second application, we determine local stress concentrations near the surface of a given section of a mountain range that are due to gravity acting on the latter. The resulting data may be used to localize potential rock failure and may thus help to improve the understanding of the principles of erosion.

REFERENCES

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