Recent advances in Hierarchical Model (HiMod) reduction

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HiMod is a model reduction technique to describe phenomena with a dominant dynamics,
possibly featuring local relevant transverse components, like for blood flow in arteries, water in rivers, oil in pipes. In particular, HiMod obtains surrogate models by resorting to a different discretization of the full problem along the main and the transverse dynamics, respectively.
Originally [1,2], the mainstream is tackled by affine finite elements while modal approximation
solves the transverse directions. The rationale is that relatively few modes are enough to capture
the transverse dynamics of interest with an overall reduction of computational costs.
Numerical results show that the approach is versatile and effective, possibly coupled with modal/nodal adaptivity in the transverse/axial direction, respectively [3].

In this presentation, we focus on the most recent advances in the HiMod setting:

(i) HiMod modeling of fluids in pipes;
(ii) extension of the HiMod approach to a parameter-dependent setting;
(iii) generalization of the HiMod procedure to curved domains.

As for (i), we have recently generalized the HiMod reduction procedure to 3D cylindrical domains and to the Stokes equations. Blood flow modeling in arteries is the reference application.
Selection of the modal basis is crucial in this context. We need a modal basis that automatically includes the boundary conditions assigned on the lateral surface (the wall of the arteries), not necessarily homogeneous Dirichlet data (in view of fluid-structure interaction problems).

(ii) To tackle the possible dependence of the HiMod approximation on a set of parameters, we propose a new methodology, called HiPOD. We aim at merging the reliability of a HiMod approximation with the computational efficiency characterizing the well-known Proper Orthogonal
Decomposition (POD). In particular, we propose two different approaches;
the first one is based on a standard POD projection; the second one adopts a two-level POD procedure based on interpolation. In the first case the problem is parametrized by problem coefficients and boundary data, in the latter one we focus on the problem coefficients only.

Concerning (iii), we have recently combined the HiMod reduction technique with the IsoGeometric Analysis (IGA), simply by replacing the finite element discretization along the mainstream with an isogeometric approximation.
The IGA intrinsic formulation allows a simpler and more effective geometrical modeling
when curved domains are considered. In particular, with a view to practical

bioengineering applications, centerlines of the arteries are obtained in a straightforward way via splines or NURBS.

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