Large Time-Step Explicit Integration Methods

for Problems Dominated by Advection

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ABSTRACT

One of the main drawbacks of the explicit integration using eulerian formulation is the restricted stability of the solution with both: the time steps and with the spatial discretization. For the case of the incompressible Navier-Stoke equations, it is well known that the time step to be used in the solution of the momentum equations is stable only for time step smaller than two critical values: the Courant-Friedrichs-Lewy (CFL) number and the Fourier number. The first one is concerning with the advective terms and the second one with the diffusive ones. Both numbers must be less than one to have stable algorithms. For problems dominated by advections like high Reynolds number flows, the condition CFL<1 becomes crucial and limit the use of explicit method or outdistance it to be efficient.

The possibility to perform parallel processing and the recent upcoming of new processors like GPGPU increase the possibilities of the explicit integration in time due to the facility to parallelize explicit methods having results with speed-up closed to one.

Although the incompressible condition cannot be solved explicitly, except introducing a small compressibility in the flow, the momentum conservation equations with an explicit integration together with a parallel processing may reduce drastically the computing time to solve the whole problem provided that a large time step may be preserved independently to the discretization in space. This presentation is concerned with this objective: to perform explicit integration of the momentum equations without the CFL<1 restriction.

The big advantage of the explicit integration is the simplicity and the scalability in parallel processors. Until now, the big disadvantage was to be conditional stable for relative small time steps and also to have stability with spatial discretization dependency. We will present here a pressure-segregation method with an explicit time integrator that allows large time steps independent of the spatial discretization having equal or better precision that an implicit integration.

The idea is to use the information we have at time $t = t^n$ in the velocity streamlines as well as in the acceleration streamlines to update the particle position as well as the velocity in a lagrangian frame. The method may be used with moving or fixed meshes.