Efficiency of high-order elements for continuous and discontinuous Galerkin methods

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ABSTRACT

In recent years, there has been an interest and no minor discussions on the benefits and utility of high-order methods compared to low order-ones. In fact, the interest is concentrated in comparing the efficiency of linear versus high-order elements (in fact, higher than one), see for instance [1,2,3,4]. Here, in order to compare high versus low-order approximations, in a manner which is implementation and hardware independent, operation count is proposed. Element (local) and global operations are considered, as well as different Galerkin methods and interpolation degrees. All major cost contributors are considered: solving the global system, generating the element matrices, removing the inner degrees of freedom (DOF), i.e. static condensation, and, of course, their recovery. This is done for 2D and 3D meshes consisting of *simplices* (triangles and tetrahedra) and *parallelotopes* (quadrilaterals and hexahedra). Standard hypotheses for this kind of analysis are employed: of large (boundary influence is negligible) uniform structured mesh, smooth solution (i.e. bounded with bounded derivatives) and such that the approximation error is controlled by the interpolation one. This allows to estimate the ratio between order-one and order-p elements for a given error tolerance, which is a key issue when comparing high versus low-order elements.

The results show that for a given accuracy there is an optimal choice of the approximation order p to minimize the cost. For solving the global system with *engineering accuracy* (usually two significant digits), high-order methods are more efficient in a wide range of p. This is also the case for element (local) operations when straight-sided elements or tensorial basis [1] are used. These results clearly refute the conclusion stated in [2] that it is difficult to improve on linear elements.

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