Fixed Mesh Methods in Computational Mechanics

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ABSTRACT

In many coupled problems of practical interest the domain of at least one of the problems evolves in time. The Arbitrary Eulerian Lagrangian (ALE) approach is a tool very often employed to cope with this domain motion. However, in this work we aim at describing numerical techniques that allow us to use a *fixed* mesh for the approximation of moving boundary problems, particularly using the finite element approach. This type of formulations is often termed embedded or immersed boundary methods. Emphasis will be put in describing a particular version of the ALE formulation using fixed meshes that we have developed, and that we call fixed-mesh ALE method (FM-ALE) [1].

In the classical ALE approach, the mesh in which the computational domain is discretized is deformed. This is done according to a prescribed motion of part of its boundary, which is transmitted to the interior nodes in a way as smooth as possible so as to avoid mesh distortion. The FM-ALE formulation has a different motivation. Instead of assuming that the computational domain is defined by the mesh boundary, we assume that there is a function that defines the boundary of the domain where the flow takes place. We will refer to it as the *boundary function*. When this boundary function moves, the flow domain changes, and that must be taken into account at the moment of writing the conservation equations that govern the flow, which need to be cast in the ALE format. However, our purpose here is to explain how to use always a background fixed mesh.

Other possibilities to use a single grid in the whole simulation can be found in the literature. They were designed as an alternative to body fitted meshes and can be divided into two main groups, corresponding in fact to two ways of prescribing the boundary conditions on the moving boundary:

- Force term. The interaction of the fluid and the solid is taken into account through a force term, which appears either in the strong or in the weak form of the flow equations. Among this type of methods, let us cite for example the Immersed Boundary method as a variant of the Penalty method, where punctual forces are added to the momentum equation, and the Fictitious Domain method, where the solid boundary conditions are imposed through a Lagrange multiplier.
- Approximate boundary conditions. Instead of adding a force term, these methods impose the boundary conditions in an approximate way once the discretization has been carried out, either by modifying the differential operators near the interface (in finite differences) or by modifying the unknowns near the interface.

REFERENCES

[1] R. Codina, G. Houzeaux, H. Coppola-Owen and J. Baiges, “The Fixed-Mesh ALE approach for the numerical approximation of flows in moving domains”, Journal of Computational Physics, Vol. **228**, pp 1591-1611 (2009).