Coupled boundary elements and finite elements - an efficient approach for simulating multiphysical systems in additive manufacturing.

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ABSTRACT

Additive manufacturing has been developing steadily in recent years. Innovation in material engineering, design and production techniques require efficient numerical methods for simulating the process in order to broaden comprehension of the multiphysical phenomenons involved. Hence, the amount of material, time and energy could be potentially decreased for existing processes, as well as the effort of experimental work to optimize the parameters could be reduced for future improvements. To achieve this, it is necessary to consider various fields of numerical analysis like fluid mechanics, fluid-structure interaction, thermomechanical contact, heat conduction, solidification, elastic deformation and electromagnetic interaction. Therefore, in an attempt to encompass all of those phenomenons efficiently in one numerical approach, the coupling between boundary element and finite element is introduced under the scope of this work.

The boundary element method (BEM) by itself is an approach for solving linear partial differential equations. The main idea is to express the unknown solution inside of a domain for given partial differential equation in terms of the boundary distributions, thereby reducing the dimensionality of the problem by one order. Due to this, potential problems like heat conduction and electromagnetic interaction governed by the Laplace equation or linear elasticity and creeping flow problems described by biharmonic equation can be solved much more efficiently than in the case of the finite element method (FEM) as the discretization of the domain is not necessary.

In order to simulate the molten material interface or liquid droplets governed by their surface tension, the FE formulation for liquid membranes is used based on [1, 2]. Membranes are modeled using Kirchhoff-Love theory resulting in a rotation-free description of the problem. Kirchhoff-Love membranes benefit from C^1-continuous shape functions to approximate the solution. To provide those, the isogeometric discretization [3] is employed as it provides a higher continuity then classical approaches.

Combining both methods, BE for the external and internal field and FE for the interface, into one coupled system results in an efficient approach for solving complex multiphysical problems solely based on the surface discretization and therefore reducing the computational effort as well as the mesh generation complexity. The coupled formulation is illustrated by several numerical examples.

REFERENCES

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